# HISTORY OF THE THEORIES OF VALUE AND PRICE - PART II ${ }^{\text {i }}$ 

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## 1. Jean-Baptiste Say (1767-1832)

Say became interested in economics from reading Adam Smith's Wealth of Nations (1776). Some historians of economic thought place Say simply as a popularizer of the theory of the classics, as someone who ordered and organized the ideas of the classics. While this is true, Say also has important theoretical contributions that, at some points, improve the theory of the classics. ${ }^{1}$ While he did not state it with complete clarity, Say tried to refute the cost production theory of the classics. He pointed out, clearly but imprecisely, that first, there is the value of things, and then there is the price. He argued that people incur costs because they first give value to final goods. Anyway, as we will see, Say was not entirely precise. His ideas about value and price can be found: (1) in his book Traite d'Economie Politique first published in $1803^{2}$ (27 years after Wealth of Nations and 14 years before Ricardo's Principles), (2) in his letters to Ricardo and Malthus, (3) in the notes he adds in the French edition of Ricardo's Principles and (4) in his A Cathechism of Political Economy.

In his Traite Say (1880, p. 285) addresses the issue of value as follows:
Valuation is vague and arbitrary, when there is no assurance that it will be generally acquiesced in by others. The owner of the house may reckon it worth 4500 dollars, while an indifferent person would value it at no more than 3500 dollars, and probably neither would be right. But if another, or a dozen other persons be willing to give for it a specific amount of other commodities, say 4000 dollars, or 4000 bushels of wheat, we may conclude the estimate to be a correct one. A house that will fetch 4000 dollars in the market is worth that sum

The paragraph shows quite a bit of influence from the scholastics, especially the concept of "generally acquiesced." In any case, he does not make a clear separation between value and price. On the one hand, there is a subjective burden when it states that valuation is vague and arbitrary and that different people can value goods differently. But simultaneously, the "common estimate" sets a target value (price). The following paragraph states the objective criterion (Say, 1880, p. 285):

The only fair criterion of the value of an object is, the quantity of other commodities at large, that can be readily obtained for it in exchange, whenever the owner wishes to part with it; and this, in all commercial dealings, and in all money valuations, is called the current price.

But Say (1880, p. 285) immediately introduces a clearly subjective aspect to the analysis:
What is it, then, that determines this-current price of commodities?
The want or desire of any particular object depends upon the physical and moral constitution of man, the climate he may live in, the laws, customs, and manners of the particular society, in which he may happen to be enrolled. He has wants, both corporeal ana intellectual, social and individual; wants for himself and for his family. His bear-skin and reindeer are articles of the first necessity to the Laplander; whilst their very name is unknown to the lazzarone of Naples, who cares for nothing in the world if he get but his meal of macaroni. In Europe, courts of justice are considered in dispensable to the maintenance of social union; whereas the Indian

[^0]of America, the Tartar, and the Arab, feel no want of such establishments. It is not our business here to inquire, wherein these wants originate; we must take them as existing data, and reason upon them accordingly.

Again, this paragraph seems to have a scholastic influence by saying that utility ultimately determines prices. It is worth remembering that the classics made it clear that goods must have value to have a price, even if they did not put enough emphasis on this point. As you can see, Say very imprecisely tried to emphasize the importance of necessity and desire in determining the price of goods. In the following paragraph, Say (1880, p. 287) tries to refute Ricardo by clarifying that it is not costs that determine the value of things but, on the contrary, it is the value that determines costs:

> Thus it is obvious, that the current value of productive exertion is founded upon the value of an infinity of products compared one with another; that the value of products is not founded upon that of productive agency, as some authors have erroneously affirmed; ${ }^{3}$ and that since the desire of an object, and consequently its value, originates in its utility, it is the ability to create the utility wherein originates that desire, that gives value to productive agency; which value is proportionate to the importance of its co-operation in the business of production, and forms, in respect to each product individually, what is called, the cost of its production.

We see that Say is clear in that costs do not set the value of goods; value determines cost (he does so imprecisely, as he uses the word value with double meaning continuously). Say (1880, pp. 290-291) makes the difference clearer in the following paragraph:

The utility of an object, or, what is the same thing, the desire to obtain it, may possibly be unable to raise its price to a level with its cost of production. In this case it is not produced, because its production would cost more than the product would be worth.

Anyway, Say (1880, p. 298) concludes, like the English classics, that prices tend to match production costs, but making it clear that it is utility and not costs that determine prices.

An ell of broad-cloth, that has, in the production, required the purchase of productive agency at the price of 8 dollars, will have cost that sum in the manufacture; but if three-fourths only of that productive agency can be made to suffice for its production; if, supposing one kind of productive agency only to be requisite, 15 instead of 20 days' labour of a single workman be enabled to complete the product, the same ell of broad-cloth will cost 6 dollars to the producer, at the same rate of wages. In this case the current price of human productive agency will have remained the same, although the cost of production will have varied in the ratio of the difference between 6 dollars and 8 dollars. But, as this difference in the relation between the cost of production and the current price of the product holds out a prospect of larger profit than ordinary in this particular channel, it naturally attracts a larger proportion of productive agency, the exertion of which, by enlarging the supply, reduces again the current price to a level with the bare cost of production.

The reasoning of the classics and that of Say are remarkably similar. The main difference is in the emphasis the French economist places that it is utility and not costs that set the price of products and that these occur if the valuation or utility raises the price enough to cover production costs. Say (1821) himself explains the difference in a letter addressed to Malthus as follows:
[...] the Price of every thing being its exchangeable value estimated in money, that there is no other price in political economy than market price: that that which Smith calls the natural price, is no more natural than any other; it is the cost of production; it is the market price of productive services.
${ }^{3}$ Say cites Ricardo in a footnote.

But Say's most relevant contribution concerning value and price is to separate the business activity from the owner of the capital. For the classics, the capitalist was also the entrepreneur; they did not separate between savers or capitalists and entrepreneurs. Although imperfectly, this separation of functions performed by Say allowed him to solve the vicious circle in which, as we saw, the classics had incurred. In a letter to Malthus, Say (1821, pp. 13-14) explains the point as follows:

To obtain a better view of the operations of industry, capital, and land, in the work of production, I personify them: and I discover that all these personages sell their services, which I call productive services, to a speculator, who may be either a trader, a manufacturer, or a farmer. This speculator having purchased the services of a landed estate, by paying a rent to the proprietor; the services of a capital, by paying interest to the capitalist; and the industrious services of workmen, factors, agents of whatever description, by the payment of salaries; consumes all these productive services, annihilates them, and out of this consumption comes a production which has a value.
The value of this production, provided it be equal to the costs of production, that is to say, to the sum necessarily advanced for all the productive services, suffices to pay the profits of all those who have concurred directly or indirectly in this production. The profit of the speculator on whose account this operation has been effected, deducting the interest of which he may have employed, represents the remuneration for his time and talents; that is to say, his own productive services employed in his own behalf. If his abilities be great, and his calculations well made, his profit will be considerable. If instead of talent he evinces inexperience in his affairs, he may gain nothing; he may very probably be a loser. All the risks attach to the speculator; but on the other hand he takes the advantage of all the favourable chances.

In his Cathechism of Political Economy, Say (1821, pp. 5-7) addresses the theory of value and price in the form of dialogue, making it clear again that it is utility that determines the value of things and that price depends on that value:

What do you understand by the world products?
I understand all those things to which men have consented to give value.
How is value given to a thing?
By giving it utility.
How is the utility of a thing the cause of its having a value?
Because persons are then to be found who are in want of this thing; they desire to have it from those who produce it. These, on their side, will not part from it until they are paid the expenses they have beet at in producing it, including their profits. The value of the thing is established by the result of this opposition between the producer and the consumer.
But there are many things of great utility, and no value, as water. Why have they no value?
Because nature gives them gratuitously, and without stint, and we are not obliged to produce them. If a person was able to create water, and wished to sell it, no one would buy it, because it could be got at the river for nothing. Thus all the world enjoy these things, but they are not riches to any body. If all things that men could desire were in the same case, no one would be rich, but no one would be in want of riches, since each could enjoy all things at his pleasure.
But this is not the case: the greater part o things which are necessary and even indispensable to us, are not given to us gratuitously and unlimitedly. Human industry much, with pains and labour, collect, fashion and transport them.
They have become products. The utility, the faculty they have acquired of being serviceable, gives them a value and this is riches.
When once riches are thus created they may be exchanged for other riches, other values, and we may procure the products which we want in exchange of those we can spare. We have seen in the preceding chapter how money facilitates this exchange.
I now conceive how products alone are riches; but their utility does not appear to be the only cause which gives them value; for there are products, such as rings and artificial flowers, which
have value but no utility.
You do not discover the utility of these products because you call only useful that which is so to the eye of the reason, but you ought to understand by that word whatever is capable of satisfying the wants and desires of man such as he is. His vanity and his passions are to him wants, sometimes as imperious as hunger. He is the sole judge of the importance that things are of to him, and of the want he has of them. We cannot judge of it but by the price he puts on them. The value of things is the sole measure of their utility to man. It is enough for us to give them utility in his eyes in order to give them a value. Now that is what we call to produce, to create products.
Recapitulate what you have said.
Give to any thing, to a material which has no value, utility, and you give it a value; that is, you make a product of it, you create wealth.

Obviously, in the quote, for Say, the word value means price while utility means use value. As can be seen, the causal relationship between utility and value (or between value and price as defined in the first part of this article) is noticeably evident. This relationship is even more apparent in the following quote (Say, 1821, pp. 26-29 Catechism of Political Economy):

We have seen how utility is given to things: we have seen that utility gives them value; how is that value fixed, the amount of which constitutes riches?
The utility which the things have acquired, causes them to be sought after, to be wanted; a price is offered for them; and when this price is sufficient to defray the expenses which their production would cost, they will be produced. [...]
I see what is the cause of the demand and of the payment for productive services: what is it that limits this demand?
The property of the consumers, or of those who desire to use the product. There would be no bounds to the demand for any useful thing if it was not to be paid for. There is no other effective demand than that which is accompanied by the offer of a price: and it is this price which in paying for the product pays at the same time for the services which were necessary to its production.
What happens when the price of the product is not sufficient to pay the charges of production?
Then the producers will not exchange their productive services for the price of the product;
and the production does not take place. ${ }^{4}$
What happens when the price of the product is more than enough to pay the charges of production?
The producers of this kind of product become more numerous, and their competition will cause the price of the production to fall. [...]
Who is that hires the labour of the one, the capital or the land of the others?
It is an undertaker of industry who unites all these means of production, and who finds in the value of the products which results from them, the re-establishment of the entire capital he employs, and the value of the wages, the interest and the rent which he pays, as well as the profits belonging to himself.
What happens when the value of the products he has created is not sufficient for all that?
He loses, if he has any thing to lose: or if he has nothing, those who have given him their confidence.

Say managed to break, perhaps with several inaccuracies, the vicious circle of the classics. The introduction of the speculator (or entrepreneur) allowed him to see with clarity that the one who decides the production costs is this agent, who interprets what price the good will have. However, he did not formalize what the Austrian school economists would call the theory of imputation a few years later.

## 2. The founders of marignalism

[^1]Marginal utility theory meant a real revolution for economic theory. Marginalism focuses on the theory of value and the analysis of income, costs, and production. The theory of marginal utility made it possible to explain with much more clarity and precision the determination of the value and price of goods. However, there are still confusing points that I would like to address in this Part II of this work.

In Part I, we saw in detail the evolution of the theory of value and price from Aristotle to the classics. We also saw that the classics did not have a theory of value (use value); what is referred to as the classical theory of value is a price theory. The price theory of the classics was based on the costs of production. According to the economists of this school, costs determine the price (value) of goods in the long run. This reasoning made them fall into a vicious circle since costs are prices. When explaining the determination of costs, they did so in terms of prices. In short, for the classics, costs determined prices, and prices determined costs. These economists could never solve this problem because they did not have a theory of value. In the previous section, we saw that Jean-Baptiste Say had largely solved the problem of circular reasoning of the classics. Marginal utility theory will make it possible to refine Say's analysis, especially the Austrian school of economics approach.

The discovery of marginal utility theory is attributed to three thinkers: William S. Jevons (1835-1882), Carl Menger (1840-1921), and Léon Walras (1834-1910). However, there were certain precursors, such as AntoineAugustin Cournot (1801-1877), ${ }^{5}$ Jules Dupuit (1804-1866), ${ }^{6}$ and Hermann Heinrich Gossen (1810-1858). ${ }^{7}$ In any case, Jevons, Menger, and Walras systematized and perceived the revolutionary importance of marginal utility theory for the theories of value and price. Although, as we will see, with several errors, they criticized the theory of the classics and contrasted it with that of marginal utility.

The vast majority of historians of economic thought claim that Jevons, Menger, and Walras came independently and virtually simultaneously to the same conclusion. The differences usually pointed out are methodological: whether they used mathematical tools, whether they were rigorous, etc. However, the main point of this Part II is to show that this conclusion is superficial. There are some differences in how the theory is explained and formalized. Still, fundamentally there are particularly crucial differences in the application. At the time of applying the theory, the conclusions reached by the three thinkers were pretty different, especially in determining prices.

Let us first see how each of them explains the marginal theory. Chronologically the first to expound the theory was Jevons in 1871; the second, Menger, also in 1871, a little after Jevons; and the third, Walras, in 1873. However, perhaps by following a mathematical method, Jevons and Walras have more points and conclusions in common than Menger. So, to make a better horizontal comparison, we will quote Jevons, Walras, and then Menger. The three authors agree that utility is the ability of an object to produce pleasure or avoid discomfort. The law of diminishing marginal utility says that as an individual owns more units of the same good, the utility it provides diminishes (the units being of equal quality and quantity). But the marginal utility theory of value holds that the value of a good is given by the utility of the last need it satisfies. Jevons (1871, p. 44) explains the topic as follows:

A quart of water per day has the high utility of saving a person from dying in a most distressing manner. Several gallons a day may posses much utility for such purposes as cooking and washing; but after an adequate supply is secured for these uses, any additional quantity is a matter of comparative indifference. All that we can say, then, is, that water up to a certain quantity, is indispensable; that further quantities will have various degrees of utility; but that beyond a certain quantity the utility sinks gradually to zero; it may even become negative, that is to say, further supplies of the same substance may become inconvenient or harmful.

Then Jevons (1871, pp. 45-46) adds:

[^2][...] the total utility of food we eat consists in maintaining life, and may be considered as infinitely great; but if we were to subtract a tenth part from what we eat daily, our loss would be but slight. We should certainly not lose a tenth part of the whole utility of food to us. It might be doubtful whether we should suffer any harm at all.
Le thus imagine the whole quantity of food which a person consumes on an average during twenty-four hours to be divided into ten equal parts. If his food be reduced by the last part, he will suffer but little; if a second tenth part be deficient, he will fell the want distinctly; the subtraction of the third tenth part will be decidedly injurious; with every subsequent subtraction of a tenth part his sufferings will be more and more serious, until at length he will be upon the verge of starvation. Now, if we call each of the tenth parts an increment, the meaning of these facts is, that each increment of food is less necessary, or possesses less utility, than the previous one.

The two quotes from Jevons show methodological differences in exposition. In the first quote, Jevons talks about water satisfying different needs: quenching thirst, cooking, washing, etc. The additional gallons of water will meet needs of different importance. However, the reasoning is different when you turn to the example of food portions. Here the analysis is about the same need, and this causes specific problems. It is not the same to say that each additional unit has a lower utility because they satisfy needs of lesser importance than to say that they have lower utility because the same need is partially satisfied. One of the problems is what is defined as the unit of analysis. Jevons (1871, pp. 47-48) himself stated that:
$[.$.$] the division of the food into ten equal parts is an arbitrary supposition. If we had taken$
twenty or a hundred or more equal parts, the same general principle would hold true, namely,
that each small portion would be less useful and necessary than the last.

True, the way Jevons explains it, the units are arbitrary. However, there is a more objective way to define unity in economic terms: the quantity that fully satisfies the need in question. In the case of food, the unit is the amount that ends with the person's appetite; in the case of water, the unit can be a glass, a bottle, a bucket, or an ocean, depending on how much is needed to meet the need. The example of water or food divided equally seems visually appealing, but it is fallacious. Food and water have a particular characteristic that does not apply to all goods. We could think of suits, watches, computers, cars, etc., whose division into subunits is impossible, as in water or food. Would Jevons say that if we remove the screws that hold the wheel of a car, or the brake pedal, the loss of utility is minimal?

Jevons' reasoning is a good example of how his eagerness to reach a continuous function to draw derivatives made him lose analytical rigor. He tried to force the use of mathematics in economic theory, and the result was not good.

Marginal utility theory based on an ordinal needs scale gives a more general and accurate explanation. Let us go back to the example of the automobile; is the unit several cars of identical characteristics? Or can we divide a car into equal minor parts? It seems evident that the unit is the car. But then we might ask: why would a person want a second car exactly like the one they already have? There seems to be no other answer than to satisfy another need of lesser importance, such as having a spare vehicle in case the one you use breaks down (also must be the same because this person does not support traveling in a different car in quality, model, color, etc.) We can give another example. If a person has a TV, why would he want another similar TV? The second TV must satisfy a different need; the same goes for a third, fourth, etc.

Walras (1874, pp. 461-463) followed the same analytical framework as Jevons, as we can see in the following quote:

We may say in ordinary language: "The want which we have for things, or the utility which things have for us, diminishes gradually as consumption increases. The more a man eats, the less hungry he is; the more he drinks the less thirsty, at least in general and apart from certain deplorable exceptions. The more hats and shoes a man has, the less need he has of a new hat or a new pair of shoes; the more horses he has in his stables, the less effort he will make to procure another horse, provided we neglect impulsive acts which our theory may ignore except when accounting for special cases." But in mathematical terms, we say: "The intensity
of the last want satisfied is a decreasing function of the quantity of the commodity consumed"; and we represent these functions by curves [...]

Walras is more abstract than Jevons in his analysis. Because he always brings his explanation into the mathematical vocabulary, there is not much more to quote. Once he explained that additional units of the same product provide a decreasing utility, he reduced this postulate to a mathematical vocabulary and continued his analysis in these terms.

While Menger's analysis has some points in common with Jevons and Walras, the Austrian economist emphasized a scale of preferences to explain why the usefulness of a good falls as you have more units. The following quote shows that Menger (1871, p. 124) has a different approach with the same example of food. Although with some imprecision, he is saying that the consumer values additional amounts of food less because they will meet other types of needs other than hunger, such as health.

> The lives of men depend on satisfaction of their need for food in general. But it would be entirely erroneous to regard all the foods they consume as being necessary for the maintenance of their lives or even their health (that is, for their continuing wellbeing). Everyone knows how easy it is to skip one of the usual meals without endangering life or health. Indeed, experience shows that the quantities of food necessary to maintain life are only a small part of what well-to-do persons as a rule consume, and that men even take much more food and drink than is necessary for the full preservation of health. Men consume food for several reasons: above all, they take food to maintain life; beyond this, they take further quantities to preserve health, since a diet sufficient merely to maintain life is too sparing, as experience shows, to avoid organic disorders; finally, having already consumed quantities sufficient to maintain life and preserve health, men further partake of foods simply for the pleasure derived from their consumption.

We can see that, in the case of Menger, unlike Jevons and Walras, the explanation of the decreasing marginal utility is based on different needs and not on the same one. Food serves to preserve life, health, and pleasure. Jevons and Walras developed their examples based on the same need: hunger. Anyway, Menger then develops a confusing table that we reproduce below:

| I | II | III | IV | V | VI | VII | VIII | IX | X |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |  |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |  |  |
| 5 | 4 | 3 | 2 | 1 | 0 |  |  |  |  |
| 4 | 3 | 2 | 1 | 0 |  |  |  |  |  |
| 3 | 2 | 1 | 0 |  |  |  |  |  |  |
| 2 | 1 | 0 |  |  |  |  |  |  |  |
| 1 | 0 |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |

Roman numerals stand for the needs of different hierarchies satisfied by various goods. And Arabic numerals represent the degree to which each consumer's need is met. The confusion arises at the following point: if, for example, 1 is food, it is unclear whether the first unit that gives a satisfaction of 10 satisfies the same need or a different one than the second unit that gives a satisfaction of 9. At this point, Menger is very imprecise. Economists of the Austrian school are characterized, or differentiated from, conventional economic theory by having developed an ordinal and non-cardinal analysis of individual valuations.

The theory of marginal utility based on an ordinal scale (of preferences) is more precise and general than that based on a single need for the inaccuracies that it generates, as we saw earlier. Later we will analyze the implicit errors that indifference curves have due to this problem.

Despite these differences, the three economists came to the same conclusion: the value of goods is given by the utility of the last unit consumed, or marginal utility. None of these three economists used the term marginal utility. Jevons (1871, p. 47) distinguished between total utility, degree of utility, and final degree of utility. The total utility is the sum of the utility of each of the units consumed. In Jevons' example, total utility is the utility of the first bite of food, plus the utility of the second, plus that of the third, etc. Jevons (1871, p. 51 italics in original) defines the degree of usefulness as follows:

The degree of utility is, in mathematical language, the differential coefficient of $u$ [utility] considered as a function of $x$, and will itself be another function of $x$ [quantity].

But immediately, he adds what would be the definition of marginal utility, the final degree of utility (italics in original):

> We shall seldom need to consider the degree of utility except as regards the last increment which has been consumed, or, which comes to the same thing, the next increment which is about to be consumed. I shall therefore commonly use the expression final degree of utility, as meaning the degree of utility of the last addition, or the next possible addition of a very small, or infinitely small, quantity to the existing stock.

Jevons (1871, p. 53 italics in original) emphasizes the importance of this concept for the economy as follows:

The variation of the function expressing the final degree of utility is the all-important point in economic problems. We may state as a general law, that the degree of utility varies with the quantity of commodity, and ultimately decreases as that quantity increases. No commodity can be named which we continue to desire with the same force, whatever be the quantity already in use or possession.

Jevons (1871, p. 59) expresses the principle that consumers maximize their usefulness when the marginal utilities of diverse needs are equalized.

Let $s$ be the whole stock of some commodity, and let it be capable of two distinct uses. Then we may represent the two quantities appropriated to these uses by $x_{1}$ and $y_{1}$, it being a condition that $x_{1}+y_{1}=s$. The person may be conceived as successively expending small quantities of the commodity; now it is the inevitable tendency of human nature to choose that course which appears to offer the greatest advantage at the moment. Hence, when the person remains satisfied with the distribution he has made, it follows that no alteration would yield him more pleasure; which amounts to saying that an increment of commodity would yield exactly as much utility in one use as in another.

Just as Jevons called what we now call marginal utility the final degree of utility, Walras used the word rareté. He defines the word in two distinct parts of his book: (1) "If, now, we let the term rareté designate the intensity of the last want satisfied [...], rareté increases as the quantity possessed decreases, and vice versa" (Walras, 1874, pp. 119-120) (2) "I call the intensity of the last need satisfied rareté" (Walras, 1874, p. 163) The English called it the final degree of utility, the Germans Grenznut-zen Walras (1874, p. 125) comes to the same conclusion as Jevons regarding the maximization of consumer utility:

Given two commodities in a market, each holder attains a maximum satisfaction of wants, or maximum effective utility, when the ratio of intensities of the last wants satisfied [by each of these goods], or the ratio of their raretés, is equal to the price. Until this equality has been reached, a party to the exchange will find it to his advantage to sell the commodity the rareté of which is smaller than its price multiplied by the rareté of the other commodity and to buy the other commodity the rareté of which is greater than its price multiplied by the rareté of
the first commodity.
It is interesting to note that, after this conclusion, Walras explains what happens when commodities are not perfectly divisible (the utility curve is not continuous). The above conclusion modifies it by saying that consumers reach the maximum of well-being not when the raretés divided by their prices are equalized but when "[...] very nearly equal [...]" (p. 129) to this equality.

Menger (1871, pp. 131-132) develops the point as follows:
If a quantity of goods stands opposite needs of varying importance to men, they will first satisfy, or provide for, those needs whose satisfaction has the greatest importance to them. If there are any goods remaining, they will direct them to the satisfaction of needs that are next in degree of importance to those already satisfied. Any further remainder will be applied consecutively to the satisfaction of needs that come next in degree of importance. [...]
Hence the value to this person of any portion of the whole available quantity of the good is equal to the importance to him of the satisfactions of least importance among those assured by the whole amount and achieved with an equal portion.

With the following example, Menger (1871, p. 132) finishes clarifying his conclusion:
Suppose that an individual needs 10 discrete units (or 10 measures) of a good for the full satisfaction of all his needs for that good, that these needs vary in importance from 10 to 1 , but that he has only 7 units (or only 7 measures) of the good at his command. From what has been said about the nature of human economy it is directly evident that this individual will satisfy only those of his needs for the good that range in importance from 10 to 4 with the quantity at his command ( 7 units), and that the other needs, ranging in importance from 3 to 1 , will remain unsatisfied. What is the value to the economizing individual in question of one of his 7 units (or measures) in this case? According to what we have learned about the nature of the value of goods, this question is equivalent to the question: what is the importance of the satisfactions that would be unattained if the individual concerned were to have only 6 instead of 7 units (or measures) at his command. If some accident were to deprive him of one of his seven goods (or measures), it is clear that the person in question would use the remaining 6 units to satisfy the more important needs and would neglect the least important one. Hence the result of losing one good (or one measure) would be that only the least of all the satisfactions assured by the whole available quantity of seven units (i.e., the satisfaction whose importance was designated as 4) would be lost, while those satisfactions [...] whose importance ranges from 10 to 5 would take place as before.

Menger gives examples of cases where the amount available decreases until the individual has only one unit at his disposal.

In Part I, we saw that some thinkers had outlined a theory of value based on marginal utility, especially the Salamanca school. However, the thinkers of this school did not detach themselves from "objective" factors in their analysis that came from the strong influence of Aristotle and a confused conception of utility. For them, the use value was determined by three factors: virtuositas, which is an "intrinsic" quality of the good; wants, which is the scarcity of the good; and complacents, which is the common estimation of a good. ${ }^{8}$ Jevons, Walras, and Menger pointed out clearly and emphatically that use value is purely subjective and that there are no objective factors in its determination.

Jevons (1871, pp. 43-44) addresses the subject in a particular point: "Utility is not an intrinsic quality," in Chapter III of his book and lays out the main ideas as follows:
[...] utility, though a quality of things, is no inherent quality. It is better described as a circumstance of things arising out of their relation to man's requirements. As Senior most accurately says, "Utility denotes no intrinsic quality in the things which we call useful; it

[^3]merely expresses their relations to the pains and pleasures of mankind." We can never, therefore, say absolutely that some objects have utility and others have not. The ore lying in the mine, the diamond escaping the eye of the searcher, the wheat lying unreaped, the fruit ungathered for want of consumers, have no utility at all. The more wholesome and necessary kinds of food are useless unless there are hands to collect and mouths to eat them sooner or later. Nor, when we consider the matter closely, can we say that all portions of the same commodity possess equal utility. Water, for instance, may be roughly described as the most useful of all substances. A quart of water per day has the high utility of saving a person from dying in a most distressing manner. Several gallons a day may possess much utility for such purposes as cooking and washing; but after an adequate supply is secured for these uses, any additional quantity is a matter of comparative indifference. All that we can say, then, is, that water, up to a certain quantity, is indispensable; that further quantities will have various degrees of utility; but that beyond a certain quantity the utility sinks gradually to zero; it may even become negative, that is to say, further supplies of the same substance may become inconvenient and hurtful.

While Walras was equally explicit about the subjectivity of value, he is a little more contradictory because of his attempt to mathematically formalize some of his conclusions. It could be said that he somehow returned to the concept of complacency of the scholastics. In defending the subjectivity of value, Walras (1874, p. 146 italics in original) says:

Rareté is personal or subjective; value in exchange is real or objective. It is only with respect to a given individual that we can define rareté in terms of effective utility and quantity possessed in a manner strictly analogous to the definition of velocity in terms of distance passed over and the time taken to pass over it, so that rareté defined as the derivative of effective utility with respect to the quantity possessed corresponds exactly to velocity defined as the derivative of distance passed over to the time taken to pass over it.

However, after a defense of subjectivity, the use of mathematics made him dizzy, and he arrived at a similar conclusion to the scholastics (Walras, 1874, p. 46 italics in original):

If we are looking for something that we might call the rareté of commodity ( A ) or commodity (B), we should have to take the average rareté, which would be the arithmetical average of the raretés of each of these commodities for all parties to the exchange after the exchange was completed. This conception of an average rareté is no more far-fetched than that of an average height or an average life span in a given country.

Walras seems to have forgotten that while rareté, as he pointed out, is a subjective concept and impossible to measure, height and lifetime are objective and possible to measure. But this is not the most relevant point: the problem is that, although height or lifetime have a practical meaning, the average rareté lacks any useful meaning. The concept of average rareté serves neither to explain the use value of goods nor their exchange value; therefore, it is a totally irrelevant concept, at least for economic science. Menger (1871, p. 116) explains the subjectivity of value as follows:

Value is therefore nothing inherent in goods, no property of them, but merely the importance that we first attribute to the satisfaction of our needs, that is, to our lives and well-being, and in consequence carry over to economic goods as the exclusive causes of the satisfaction of our needs.

And after explaining that value arises from the fact that goods are necessary and scarce Menger (1871, pp. 120-121) repeats again:

Value is thus nothing inherent in goods, no property of them, nor an independent thing
existing by itself. It is a judgment economizing men make about the importance of the goods at their disposal for the maintenance of their lives and well-being. Hence value does not exist outside the consciousness of men.

While Menger emphasized more than Jevons and Walras that value is not something inherent in goods but something that is in the minds of individuals, he was not free of contradictions. The following paragraph shows a Menger (1871, p. 53) influenced by objectivism; the influence, as will be seen, seems to come from Aristotle himself: ${ }^{9}$

A special situation can be observed whenever things that are incapable of being placed in any kind of causal connection with the satisfaction of human needs are nevertheless treated by men as goods. This occurs (1) when attributes, and therefore capacities, are erroneously ascribed to things that do not really possess them, or (2) when non-existent human needs are mistakenly assumed to exist.

The most striking thing is that immediately after a paragraph where he emphatically emphasizes that value is totally subjective and that there is no objective or inherent value to goods, Menger (1871, p. 120 italics in original) states:

> Regarding this knowledge, however, men can be in error about the value of goods just as they can be in error with respect to all other objects of human knowledge. Hence they may attribute value to things that do not, according to economic considerations, possess it in reality, if they mistakenly assume that the more or less complete satisfaction of their needs depends on a good, or quantity of goods, when this relationship is really nonexistent. In cases of this sort we observe the phenomenon of imaginary value.

These paragraphs clash strongly after emphasizing the subjective character of value. The contradiction could have the following explanation: all action is towards the future, which implies an ex-ante or expected result and an ex-post or real result. When men act, they do so based on ex-ante results, expecting specific effects. Once the action is finished, the ex-post (real) results appear. Someone can buy a good, misbelieving that it will have a beneficial effect, for example, drinking a lot of whisky to calm a headache. As we will see below, Menger, unlike Jevons and Walras, distinguished between the concepts of "price" and "expected price," which helped him to develop a more precise theory than his colleagues. That is why Menger's contradiction concerning value can be explained as an unhappy attempt to distinguish between ex-ante and ex-post values. To explain the formation of prices, the only thing that counts is the ex-ante value; people a2edsct based on expectations that are later corrected or not based on ex-post results. In other words, action may involve an error, which does not mean irrationality. The one who dances a dance until it rains is being rational since he is associating a relationship of cause-and-effect in the dance-rain ritual.

So far, we have seen how the three founders of marginal analysis explained the determination of use value. In Part I, we saw that classical economists (like the Aristotelian tradition) distinguished between use value or simply value and exchange value or price. They never developed a theory of use value. They simply took it for granted that for any good to have a price (exchange value), it had to have use value first. In connection with this, it is important to remember that marginal utility theory is a theory that explains use value and not exchange value. It is wrong to contrast the theory of marginal value with the classic's theory of value. What needs to be done is to contrast the price theory of the marginalists with that of the classics. Still, almost all marginalists cautioned against the ambiguity of the word value. Jevons (1871, pp. 76-77) acknowledges that: "In spite of the most acute feeling of the danger, I often detect myself using the word improperly; nor do I think that the best authors escape the danger." For this reason, the words utility began to be used to designate the use value and price to appoint the exchange value.

Due to the lack of a theory of value, the classics had a flawed theory of prices. They concluded that in the long run, production costs determine final prices or, in their own terminology, the natural price was

[^4]determined by costs. The fundamental problem with the price theory of the classics is that it enters a vicious circle: they explain prices according to costs and costs according to prices. The theory of marginal utility should have served to solve this problem of the classics; however, at this point, it can be said without much margin of error that the Austrian school was the only one that solved the problem; in England, Jevons got closer to the solution.

When the three economists move from the theory of marginal utility to that of exchange, they divide the analysis into two cases: (1) an exchange between two people who each own a stock of different goods. In this case, there is no production (supply is a given); (2) the supply is subject to a production process. In the first case, there are no production costs; in the second, there are production costs. Regarding the first, the three arrive at the same conclusion using similar examples: two people will make an exchange if the marginal utility of the good they receive is greater than that of the one they deliver. In other words, when they value the good, they receive more than the good they provide. With the exchange, both parties have more of one of the goods and less of the other. The marginal utility of the received good, which has greater marginal utility than that which is delivered, decreases, and that of the one that is offered increases. Here we have the first difference between Jevons and Walras on the one hand and Menger on the other. For the first two, the exchange between people ceases when marginal profits are equalized. This relates to the price or quantity of a good delivered in exchange for the other. If the price is one unit for another unit, then exchange ceases when the marginal utilities of the two goods are equalized. If the price is not one-to-one, the exchange ends when the marginal utility of a good divided by its price equals the marginal utility of the other divided by its price. If the two goods are $x$ and $y$, this conclusion is expressed in the following mathematical symbology: $U_{x} x=U_{y} y$. Where $U_{x}$ and $U_{y}$ denote the marginal utilities of $U_{x}$ and $U_{y}$ respectively and $x$ and $y$ their respective quantities. This conclusion can be found in any book of contemporary microeconomics expressed as follows: $U_{x} / p_{x}=U_{y} / p_{y}$, or equally, $U_{x} / U_{y}=$ $p_{x} / p_{y}$. If $x$ and $y$ are equal to one, the exchange ends when the marginal profits of the goods are equalized.

Jevons and Walras come to this conclusion because they assume that the goods exchanged can be divided infinitesimally, hence continuous functions and corresponding derivatives. Jevons (1871, p. 95 italics in original) expresses this conclusion as follows:

> The keystone of the whole Theory of Exchange, and of the principal problem of Economics, lies in this proposition - The ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed.

For his part, Walras (1874, p. 145 italics in original)says:

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Current prices or equilibrium prices are equal to the ratios of the raretés.
In other words:
Values in exchange are proportional to the raretés.
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Jevons and Walras' conclusions about when the exchange ceases are identical. Menger does not come to precisely the same conclusion. As long as the marginal utility of the received good is greater than that of the offered one, well-being improves. Menger (1871, p. 187) explains the cessation of the exchange as follows:

This limit is reached when one of the two bargainers has no further quantity of goods which is of less value to him than a quantity of another good at the disposal of the second bargainer who, at the same time, evaluates the two quantities of goods inversely.

Unlike Jevons and Walras, Menger did not assume the perfect divisibility of the goods exchanged and based his analysis on an ordinal scale. Therefore, his conclusion was different: exchange ceases when what is received is no longer valued more than what is delivered. Jevons and Walras came to the same conclusion when they added the assumption of continuity in the function, taking the case of continuity as the most general and that of discontinuity as a less general one. It does not require much thought to see that Menger's conclusion is more precise and general. Neither goods are infinitely divisible, nor do people value in infinitesimal amounts. Both units and valuations are made in discrete quantities, and the basis of the exchange is made with ordinal and
non-cardinal scales, as Jevons and Walras assume. If we want to be rigorous as Jevons and Walras intended when using mathematical symbology, we must conclude that marginal utilities can never be equalized. The cessation of exchange is one step before reaching this equality, especially if we want to draw rigorous conclusions. It is paradoxical that by using mathematics, Jevons and Walras, who wanted to make economics rigorous, were less precise than Menger. This theoretical error of Jevons and Walras is passed down from generation to generation of economists.

Our next step is to see how these economists used marginal utility theory to explain the determination of prices if production is needed because there is no given stock. Classical economists distinguished between the market price and the natural price. The former is determined by supply and demand, while the latter is determined by the cost of production. For the classics, the market price tends to be equal to the natural one, which is why they concluded that costs determine final prices in the long run. Jevons refutes this theory of the classics. However, it makes the widespread mistake of assuming that the classics had a labor-based theory of exchange value, more commonly called the labor theory of value. As we saw in Part I, this is wrong, even in the confusing case of Ricardo (Marx's case may be a little different). For the classics, labor was not the only determinant of costs, except in the case of primitive societies. Taking this interpretative error into consideration, Jevons (1871, p. 163) refutes the classics as follows:

The mere fact that there are many things, such as rare ancient books, coins, antiquities, etc., which have high values, and which are absolutely incapable of production now, disperses the notion that value depends on labour. Even those things which are producible in quantity of labour seldom exchange exactly at the corresponding values.

And he adds (p. 164, italics in original):
The fact is, that labour once spent has no influence on the future value of any article: it is gone and lost forever. In commerce bygones are for ever bygones; and we are always starting clear at each moment, judging the values of things with a view to future utility. Industry is essentially prospective, not retrospective; and seldom does the result of any undertaking exactly coincide with the first intention of its promoters.

However, he immediately ends up taking a detour like this (pp. 164-165, italics in original):
But though labor is never the cause of value, it is in large proportion of cases the determining circumstance, and in the following way:-Value depends solely on the final degree of utility. How can we vary this degree of utility? - By having more or less of the commodity to consume. And how shall we get more or less of it? -By spending more or less labour in obtaining a supply. According to this view, then, there are two steps between labour and value. Labour affects supply, and supply affects the degree of utility, which governs value, or the ratio of exchange. In order that there may be no possible mistake about this all-important series of relations. I will re-state it in a tabular form, as follows: -

Cost of production determines supply.
Supply determines final degree of utility.
Final degree of utility determines value.
Finally, Jevons (1871, p. 165 italics in original) adds a paragraph that implies a change of 180 degrees from the classics:

> I hold labour to be essentially variable, so that its value must be determined by the value of the produce, not the value of the produce by that of the labour.

One could conclude that, even with some degree of inconsistency, Jevons turned around the conclusion of the classics: it is not costs that determine prices but prices that determine costs.

Walras devotes a chapter (the 16th) of his book to refuting the theories of Adam Smith and Jean-Baptiste Say. According to Walras (1874, p. 201 italics in original):

The science of economics offers three major solutions to the problem of the origin of value. The first, that of Adam Smith, Ricardo and McCulloch, is the English solution, which traces the origin of value to labour. This solution is too narrow, because it fails to attribute value to things which, in fact, do have value. The second solution, that of Condillac and J. B. Say, is the French solution, which traces the origin of value to utility. This solution is too broad, because it attributes value to things which, in fact, have no value. Finally, the third solution, that of Burlamaqui and my father, A. A. Walras, traces the origin of value to scarcity ['rareté']. This is the correct solution.

At this point, Walras is misinterpreting Smith. As we have already seen, neither the Scots nor the classical had a theory of value, if by value we mean use value or utility. They had a theory of price or exchange value, but it was also not based on labor but on the cost of production. To argue that Smith had a labor theory of value is very superficial; Walras does not seem to have read The Wealth of Nations carefully. Regarding Say, Walras may not have read his Cathechism. In our quote 12, you can see how Say answers the question, right or wrong: But there are many things of great utility that have no value, like water. Why do they have no value? You may or may not agree with Say's answer, but what Walras cannot say is that his theory is very broad. In reality, Walras' own theory, as we shall see, is identical to Say's. According to Say's answer, exchange value is determined by utility and costs; Walras' theory says precisely the same in mathematical vocabulary. Finally, what he calls scarcity theory, which he attributes to Burlamaqui and his father, is confusing. A theory based exclusively on scarcity or rareté remains as broad as utility since it attributes value to things that do not have it. Some things are scarce and do not have value or more value than the less scarce, for example, airplanes with a single wing or racehorses that are missing a leg. Anyway, Walras called rareté the utility of the last satisfied need and perhaps used the word scarcity a little hastily, making his exposition contradictory when it is very likely that he meant marginal utility and not scarcity. That Walras implied this is very clear in his quote from Burlamaqui (Walras, 1874, pp. 203-204):

> One of the foundations of inherent and intrinsic price is the capacity which things have to serve our wants, our convenience and our pleasures of life; in other words, it is the utility of these things. Another foundation is their scarcity.
> When I speak of utility I mean not only real utility, but also utility that is only arbitrary or imaginary, like the utility of precious stones. It is common knowledge that a thing that is absolutely useless has no price.
> But utility alone, be it ever so real, does not suffice to give a price to things. In addition, their scarcity must be considered - that is to say, the difficulty of procuring them, so that no one can easily get as much as he wants of them. Need alone is very far from determining the price of a thing. Everyday experience shows us that those things which are the most necessary to human life are the cheapest - ordinary water, for example.
> Scarcity alone is not sufficient to give price to things either. They must be of some use as well. [...]
> To resume, all the special circumstances which cause a thing to have a high price can be brought under the heading of scarcity. Such special circumstances are, for example, the difficulty of making the thing, or its peculiar intricacy, or the unique reputation of the artisan who made it.

Burlamaqui's solution is the same as Say's, and one could also say that of the classics. Recall that the classics held that for a thing to have exchange value, it must first have use value and that the long-run price is determined by the cost of production. Therefore, they were saying, perhaps not very clearly, that the price of things is determined by the binomial utility and costs. Walras seems to have read the classics very hastily. ${ }^{10}$

[^5]Walras' explanation of Price determination does not differ too much from the classics. In his own way, Walras distinguishes between short-run prices (market prices for the classics) and long-run prices (natural for the classics). Walras' explanations rest on two laws: (1) the law of determination of equilibrium prices and (2) the law of variations of commodity prices. Walras (1874, pp. 172, 180 italics in original) his laws the following way:
[Law of determination of equilibrium prices]
We are now in a position to formulate the law of establishment of equilibrium prices in the case of the exchange of several commodities for one another through the medium of a numéraire: Given several commodities, which are exchanged for one another through the medium of a numéraire, for the market to be in a state of equilibrium or for the price of each and every commodity in terms of the numéraire to be stationary, it is necessary and sufficient that at these prices the effective demand for each commodity equals its effective offer. When this equality is absent, the attainment of equilibrium prices requires a rise in the prices of those commodities the effective demand for which is greater than the effective offer, and a fall in the prices of those commodities the effective offer of which is greater than the effective demand.
[...]
[The law of the variation of commodity prices]
Given a state of general equilibrium in a market for several commodities where exchanges take place with the aid of a numéraire, if the utility of one of these commodities increases or decreases for one or more parties, everything else remains equal, the price of this commodity in terms of the numéraire will increase or decrease.
If the quantity of one commodity in the hands of one or more holders increases or decreases, all other things remaining equal, the price of this commodity will increase or decrease.
[...]
Given several commodities, if both the utility and the quantity of one of these commodities in the hands of one or more parties or holders vary in such a way that the raretés remain the same, the price of this commodity will not change.
If the utility and the quantity of all the commodities in the hands of one or more parties or holders vary in such a way that the ratios of the raretés remain the same, none of the prices will change. Such is the law of the variations of equilibrium prices. When it is combined with the law of the establishment of equilibrium prices, we have the scientific formulation of what is known in economics as the LAW OF SUPPLY AND DEMAND.

As we can see, Walras comes to the same conclusion as the classics, except that he adds the concept of rareté (marginal utility) and the use of mathematics. Walras (1874, p. 181) marks his difference as follows:

I venture, therefore, to assert that, up to the present, this fundamental law of economics has neither been demonstrated nor even correctly formulated. And I go as far as to maintain that it is impossible either to formulate or to demonstrate the law of supply and demand or the two laws of which it is composed, without defining effective demand and effective offer, and showing their relationship to price, or without defining rareté and showing its relationship to price too. We can do this only by recourse to the language, the method and the principles of mathematics. Hence we conclude that the use of mathematics is not only possible but necessary and indispensable in the formulation of pure economics.

[^6]Even Warlas' (1874, p. 255 italics in original) conclusion that in the long run, prices are equal to costs is the same as that of the classics:

> Production in a market ruled by free competition is an operation by which services can be combined and converted into products of such a nature and in such quantities as will give the greatest possible satisfaction of wants within the limits of the double condition, that each service and each product have only one price in the market, namely the price at which the quantity supplied equals the quantity demanded, and that the selling price of the products be equal to the cost of the services employed in making them.

It can be concluded that Walras, like the classics, differentiated between a short-run price determined by supply and demand and a long-term price in which prices and costs were equalized. Suppose the short-term price, due to variations in the rareté or the quantity available, moves away from the long-term price. In that case, competition will tend to restore the long-term equilibrium by adjusting the quantities produced.

Walras (1874, p. 380) explains how equilibrium prices are determined through a process of trial and error (tátonnement):

> Every hour, nay, every minute, portion of these different classes of circulating capital are disappearing and reappearing. Personal capital, capital goods proper and money also disappear and reappear, in a similar manner, but much more slowly. Only landed capital escapes this process of renewal. Such is the continuous market, which is perpetually tending towards equilibrium without every actually attaining it, because the market has no other way of approaching equilibrium except by groping and, before the goal is reached, it has to renew its efforts and start over again, all the basic data of the problem, e.g., the initial quantities possessed, the utilities of goods and services, the technical coefficients, the excess of income over consumption, the working capital requirements, etc., have changed in the meantime.
> Viewed in this way, the market is like a lake agitated by the wind, where the water is incessantly seeking its level without ever reaching it. But whereas there are days when the surface of a lake is almost smooth, there never is a day when the effective demand for products and services equals their effective supply and then the selling price of products equals the cost of the productive services used in making them.

In Lesson 38, Walras exposes and refutes the theory of prices of classical economists in which production costs determine prices. Walras (1874, pp. 399-400) opposes this conclusion by arguing that it is just the opposite:

Nor is there any one value of costs of production, which, having itself been determined, determines in turn the selling prices of products. The selling prices of products are determined in the market for products by reason of the utility and their quantity. There are no other conditions to consider, for these are the necessary and sufficient conditions. It does not matter whether the products cost more or less to produce than the selling prices. If they cost more, so much worse to the entrepreneur - it is his loss. If they cost less, so much better for the entrepreneur - it is his gain. It is not the cost of the productive services that determines the selling price of the product, but rather the other way around. In fact, the prices of productive services are established in the market for services according to their offer by land-owners, workers and capitalists and their demand by entrepreneurs. On what does this demand depend? On the prices of the products. When the outlay on production is greater than the selling price, entrepreneurs reduce their demand for productive services and the prices of services decline. When the outlay on production is less than the selling price, entrepreneurs increase their demand for productive services and their prices of services rise. This is the way in which these phenomena are related. Any other conception of the relationship is erroneous.

In this quote, we can see that Walras practically gives a $180^{\circ}$ turn to the conclusion of the classical economists. It is not costs that determine prices but prices that determine costs. Walras' conclusion differs from
the standard one, according to which utility and costs determine prices simultaneously; there is no teleological relationship between the two variables. This generalized interpretation is justified by how Walras develops his book with intense use of linear equations where there are no ideological relations but a simultaneous determination. William Jaffé, the economist who possibly studied Walras the most (besides being his translator), states in the translator's preface to Walras (1874, p. 6) that: "[Walras] had received no academic preparation in economics at all. In theoretical economics he had but one teacher, his father. For the rest, he was self-taught; but, as we know from the Eléments, he did not depart from the classical tradition, which he criticized only to perfect and enlarge its scientific structure."

In the last edition of his Eléments Walras, he compares his theory with that of Jevons and Menger. He maintains that it has no significant differences with Jevons except that the English model applies only to the case of two goods, while its own is of a general nature. Regarding Menger, he admits that he and his disciples developed a very good theory despite not using mathematics and instead the imperfect method of words (Walras, 1874, pp. 204-207). Walras' mathematical training, like the economic one, was relatively poor. Maybe it was this ignorance that made him a little arrogant. ${ }^{11}$ His lack of mathematical preparation did not allow him to see that the equality of equations and unknowns does not guarantee the system has a solution. This error of Walras lasted for an extended period of time. Paradoxically the problem was solved through Carl Menger's son, Karl Menger, a prestigious mathematician of the Vienna circle. A student of his, Abraham Wald, solved for the first time Walras' problem. As we will see later, the reality was just the opposite of what Walras thought: mathematical language gave rise to a less precise and rigorous economic theory.

Menger has several significant differences from Jevons and Walras. By not using mathematics, he was much more rigorous and precise than his colleagues, and he did not get lost in unnecessary and inaccurate simplifications. First, it separated the concepts of price and expected price. This differentiation allowed him to better solve the vicious circle of the classics. For Menger, as for Jevons and Walras, marginal utility explains the use value of things but not the price of productive factors. Menger put much more emphasis than his two colleagues on pointing out that only the marginal utility of buyers and sellers determines prices. For Menger, as for all marginalists, the exchange of commodities occurs when each party values more the good it receives than the one it delivers or, what is equal, the marginal utility of the good received must be superior to that of the given. Thus, if buyer $A$ is willing to pay up to $\$ 100$ for good $x$, it means that the marginal utility of $x$ is greater than that of $\$ 100$ (or that of all the goods that could be bought with that $\$ 100$ ). This $\$ 100$ is the maximum limit that $A$ is willing to go to, but obviously, he would also be willing to pay less. The maximum limit is given by the marginal utility or valuation of the buyer. If seller $B$ is unwilling to sell at less than $\$ 70$, this means that for him, the marginal utility of $\$ 70$ is higher than that of the good $x$ he must deliver. In this case, the $\$ 70$ is the lower limit, but obviously, you would be willing to sell at any higher price. The seller's marginal profit or valuation determines the lower limit he is ready to sell. In this case, the price at which the transaction is made has to be between $\$ 100$ and $\$ 70$. Outside these limits, there is no possible transaction; above $\$ 100$, the buyer does not buy, and below $\$ 70$, the seller does not sell. Menger (1871, pp. 194-195) explains it like this:

By way of illustration, we will suppose that 100 units of A's grain have the same value to him as 40 units of wine. It is clear from the beginning that A will, under no circumstances, be prepared to give more than 100 units of grain for 40 units of wine in an exchange, since if he were to do so, his needs would be less well provided for after the exchange than before. He will agree to an exchange only if it enables him to make better provision for his needs than would be possible without the exchange. He will be willing to exchange his grain for wine only if he has to give less than 100 units of grain for 40 units of wine. Thus whatever the price of 40 units of wine may eventually be in an exchange of A's grain for the wine of some other economizing individual, this much is certain, that it cannot, owing to the economic position of A, reach 100 units of grain.
If A can find no other economizing individual to whom a smaller quantity than 100 units of grain has a greater importance than 40 units of wine, he will never be in a position to exchange his grain for wine. In this event, the foundations for an economic exchange of the two goods would not be present so far as A is concerned. But if A does find a second economizing

[^7]individual, $B$, to whom only 80 units of grain, for example, have a value equal to 40 units of wine, the prerequisites for an economic exchange between $A$ and $B$ are certainly present (provided the two men recognize the situation and no barriers stand in the way of execution of the exchange), and at the same time a second limit is set to price formation. If it follows from the economic situation of A that the price of 40 units of wine must be below 100 units of grain (since he would otherwise derive no economic gain from the transaction), it follows from the economic situation of B that a greater quantity than 80 units of grain must be offered for his 40 units of wine. Hence, whatever the price that is finally established for 40 units of wine in an economic exchange between $A$ and $B$, this much is certain, that it must be formed between the limits of 80 and 100 units of grain, above 80 and below 100 units.

After developing this example, Menger analyzes how the price is determined by several buyers and a single seller. He then moves on to the case of several sellers and a single buyer and ends with the case in which there are several buyers and sellers. It is assumed that each buyer and each seller demand and offer only one unit; the conclusion stays unchanged if everyone demands or sells more than one unit. In Menger's quote, the price of wine is between $\$ 100$, the maximum price the buyer is willing to pay, and $\$ 80$, the minimum the seller is willing to sell. If a second buyer now appears who is willing to pay up to $\$ 94$ units of grain for one unit of wine, the previous minimum is modified. The new limits within which supply and demand are equalized are $\$ 100$ and more than $\$ 94$. Two units are demanded at $\$ 94$ or less, and one is offered. Above $\$ 94$ and up to $\$ 100$, one unit is demanded and offered. Above $\$ 100$, the demand is zero, and one unit is offered. So, the equilibrium price, which equals supply and demand, is included, in this second case, between $\$ 100$ and $\$ 94$.

A third case is when you add a second seller instead of a buyer, so we have two sellers and a buyer. Always assuming that everyone wants to buy and sell a unit, if the second seller does not sell for less than $\$ 84$, the price must be between $\$ 80$ and $\$ 84$ units of grain. If the price is $\$ 84$, two units are offered, and one is demanded. The supply is zero at less than $\$ 80$, and the demand is one unit. Therefore, the equilibrium price must be between $\$ 80$ and less than $\$ 84$. With these three cases, Menger analyzes the general case showing that it is always marginal buyers and sellers who determine equilibrium prices. The following four examples show all the possibilities of pricing. The C's and V's stand for buyers and sellers, respectively; each can buy or sell one or more units. Thus, for example, at $\$ 98$, the quantity demanded is one, and the amount offered is nine. If the price is $\$ 40$, the quantity demanded is 6 units, and the amount offered is one. In case 1 , the equality between supply and demand occurs within the price limits of $\$ 75$ and $\$ 70$. At $\$ 75$, the quantity offered and demanded is four units; the same goes for a price of $\$ 70$. Above $\$ 75$ or below $\$ 70$, the quantities demanded and offered differ.

| Case 1 |  | Case 2 |  | Case 3 |  | Case 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | V | C | V | C | V | C | V |
| 98 | 40 | 98 | 40 | 98 | 40 | 98 | 40 |
| 90 | 60 | 90 | 60 | 90 | 60 | 90 | 60 |
| 82 | 68 | 82 | 68 | 82 | 68 | 82 | 68 |
| 75 | 70 | 75 | 70 | 75 | 70 | 75 | 79 |
| 68 | 83 | 72 | 83 | 68 | 74 | 72 | 74 |
| 60 | 84 | 60 | 84 | 60 | 84 | 60 | 84 |
| 45 | 89 | 45 | 89 | 45 | 89 | 45 | 89 |
| 30 | 90 | 30 | 90 | 30 | 90 | 30 | 90 |
| 20 | 96 | 20 | 96 | 20 | 96 | 20 | 96 |

In the first case, the marginal buyer and seller are the ones who determine the price limits. The marginal buyer is the first to withdraw if the price rises, and the marginal seller is the first to withdraw if the price falls. The marginal buyer determines the maximum limit, and the marginal seller determines the minimum limit.

In case 2 , there is a variation. The minimum limit is not set by the marginal seller but by the submarginal buyer (who is the first to buy in case the price drops). In this case, the price $(p)$ that equals the quantity demanded and offered is $\$ 75>p>\$ 72$. The maximum limit is determined by the marginal buyer and the minimum by the submarginal buyer.

In case 3 , the price that equals supply and demand is $\$ 74>p>\$ 70$. The lower limit is set by the marginal seller, and the maximum by the submarginal seller. Outside these limits, the quantities offered and demanded are not equalized.

Finally, in case 4 , the equilibrium price is $\$ 74>p>\$ 72$. The submarginal buyer and the submarginal seller set the limits. In this way, Menger shows, with greater clarity and precision than mathematical functions do, that marginal and submarginal buyers and sellers set equilibrium prices. No matter how long the list of quantities demanded and offered, the solution lies in the margin. If the price exceeds certain limits, marginal and submarginal buyers and/or sellers enter or leave the market, unbalancing the equality between supply and demand.

The theory of marginal utility served Menger to set the limits within which the market equilibrium price is established. Consequently, Menger (1871, p. 192 italics in original) rejected the Aristotelian conclusion that values are equalized in exchange and also the conclusion of the classics that the costs of production determine relative prices:

If the locks between two still bodies of water at different levels are opened, the surface will become ruffled with waves that will gradually subside until the water is still once more. The waves are only symptoms of the operation of the forces we call gravity and friction. The prices of goods, which are symptoms of an Economic equilibrium in the distribution of possessions between the economies of individuals, resemble these waves. The force that drives them to the surface is the ultimate and general cause of all economic activity, the endeavor of men to satisfy their needs as completely as possible, to better their economic positions. But since prices are the only phenomena of the process that are directly perceptible, since their magnitudes can be measured exactly, and since daily living brings them unceasingly before our eyes, it was easy to commit the error of regarding the magnitude of price as the essential feature of an exchange, and as a result of this mistake, to commit the further error of regarding the quantities of goods in an exchange as equivalents. The result was incalculable damage to our science since writers in the field of price theory lost themselves in attempts to solve the problem of discovering the causes of an alleged equality between two quantities of goods. Some found the cause in equal quantities of labor expended on the goods. Others found it in equal costs of production. And a dispute even arose as to whether the goods are given for each other because they are equivalents, or whether they are equivalents because they are exchanged. But such an equality of the values of two quantities of Goods (an equality in the objective sense) nowhere has any real existence.

Regarding the classical theory of production costs, Menger (1871, pp. 146-147, italics in original) makes the following observation:

The value an economizing individual attributes to a good is equal to the importance of the particular satisfaction that depends on his command of the good. There is no necessary and direct connection between the value of a good and whether, or in what quantities, labor and other goods of higher order were applied to its production. A non-economic good (a quantity of timber in a virgin forest, for example) does not attain value for men if large quantities of labor or other economic goods were applied to its production. Whether a diamond was found accidentally or was obtained from a diamond pit with the employment of a thousand days of labor is completely irrelevant for its value. In general, no one in practical life asks for the history of the origin of a good in estimating its value, but considers solely the services that the good will render him and which he would have to forgo if he did not have it at his command. Goods on which much labor has been expended often have no value, while others, on which little or no labor was expended, have a very high value. Goods on which much labor was expended and others on which little or no labor was expended are often of equal value to economizing men. The quantities of labor or of other means of production applied to its production cannot, therefore, be the determining factor in the value of a good. Comparison of the value of a good with the value of the means of production employed in its production does, of course, show whether and to what extent its production, an act of past human activity, was appropriate or economic. But the quantities of goods employed in the production of a good have neither a necessary nor a directly determining influence on its value.
[...]
The determining factor in the value of a good, then, is neither the quantity of labor or other goods necessary for its production nor the quantity necessary for its reproduction, but rather the magnitude of importance of those satisfactions with respect to which we are conscious of being dependent on command of the good. This principle of value determination is universally valid, and no exception to it can be found in human economy.

Like most marginalists, Menger's quote shows a great misunderstanding of what the classics were saying. As we saw in Part I, the classics did not have a theory of value (use value) but of price (exchange value). What Menger says about the classics could be refuted by the classics themselves. It is worthwhile to cite a paragraph by Ricardo (1817, p. 11), who is erroneously attached a labor-theory of value: "Utility then is not the measure of exchangeable value, although it is absolutely essential to it. If a commodity were in no way useful, -in other words, if it could in no way contribute to our gratification, -it would be destitute of exchangeable value, however scarce it might be, or whatever quantity of labour might be necessary to procure it." Marx (1867, p. 48) himself, typical representative of the so-called labor theory of value, says: "[...] nothing can have value, without being an object of utility. If the thing is useless, so is the labour contained in it; the labour does not count as labour, and therefore creates no value."

Menger lost sight of classical theory's central problem, asserting that production costs determine prices in the long run. This, in turn, led these economists into the vicious circle of claiming that costs determine prices and then that prices determine costs. As we shall see, Eugen von Böhm-Bawerk better captured the error of the classics. You cannot criticize the theory of value of the classics simply because they did not have one.

Nevertheless, Menger gave a more precise solution than Walras and Jevons to get out of the vicious circle of the classics by distinguishing between prices and expected prices. In the following quote, we can see the explanation of the determination of the price of the productive factors, goods of higher order in Menger's (1871, p. 150 italics in original) terminology:
[...] it is evident that the value of goods of higher order is always and without exception determined by the prospective value of the goods of lower order in whose production they serve. The existence of our requirements for goods of higher order is dependent upon the goods they serve to produce having expected economic character and hence expected value.
[...] therefore have the principle that the value of goods of higher order is dependent upon the
expected value of the goods of lower order they serve to produce. Hence goods of higher order can attain value, or retain it once they have it, only if, or as long as, they serve to produce goods that we expect to have value for us. If this fact is established, it is clear also that the value of goods of higher order cannot be the determining factor in the prospective value of the corresponding goods of lower order. Nor can the value of the goods of higher order already expended in producing a good of lower order be the determining factor in its present value. On the contrary, the value of goods of higher order is, in all cases, regulated by the prospective value of the goods of lower order to whose production they have been or will be assigned by economizing men.
The prospective value of goods of lower order is often-and this must be carefully observedvery different from the value that similar goods have in the present. For this reason, the value of the goods of higher order by means of which we shall have command of goods of lower order at some future time is by no means measured by the current value of similar goods of lower order, but rather by the prospective value of the goods of lower order in whose production they serve.

And he adds later (p. 151):
Hence there is no necessary connection between the value of goods of lower or first order in the present and the value of currently available goods of higher order serving for the production of such goods. On the contrary, it is evident that the former derive their value from the relationship between requirements and available quantities in the present, while the latter derive their value from the prospective relationship between the requirements and the quantities that will be available at the future points in time when the products created by means of the goods of higher order will become available. If the prospective future value of a good of lower order rises, other things remaining equal, the value of the goods of higher order whose possession assures us future command of the good of lower order rises also. But the rise or fall of the value of a good of lower order available in the present has no necessary causal connection with the rise or fall of the value of currently available corresponding goods of higher order.
Hence the principle that the value of goods of higher order is governed, not by the value of corresponding goods of lower order of the present, but rather by the prospective value of the product, is the universally valid principle of the determination of the value of goods of higher order.

In this way, Menger gave a coherent way out of the trap into which the classics had fallen. But with this, he refuted or solved the theory of prices and not the theory of value. In addition, Menger's solution is much more straightforward, elaborate, and precise than Jevons and Walras's. The introduction of expectations, which Jevons and Walras did not do, made a significant difference between the Austrians and the mathematical school, which assumed perfect knowledge on the part of economic agents.

## 3. The disseminators of marginalism. A step back

For several reasons, none of the discoverers of marginal utility theory succeeded in presenting their revolutionary ideas. In the case of Jevons, he collided with the enormous prestige that classical economics had achieved, especially with John S. Mill. Jevons was rejecting Mill's ideas and also using novel mathematical language, which at the time was considered by most economists to be inappropriate for social sciences. A similar fate was that of Walras. The use of mathematics marginalized him quite a bit from the circle of French economists and even more so because he declared himself a scientific socialist in a circle in which the liberal idea predominated. For his part, Menger wrote in an environment where historicism dominated the academic world. Historicists denied that it was possible to determine laws of universal validity in social sciences, as in natural sciences. For historicists, each country and each historical moment had its own characteristics. Like Jevons and Walras' books, Menger' was theoretical and therefore rejected in academia. None of the three authors could show the importance of marginal utility theory for economic science.

Marginal utility theory began to make its way through the followers of these economists, mainly thanks to Alfred Marshall and Eugen von Böhm-Bawerk. Still, Pareto, Cassel, and Wieser contributed to disseminating and clarifying marginal utility theory. However, they all took a step back from their respective inspirers' words.

### 3.1. Alfred Marshall (1842-1924)

Marshall must be the economist who had the most influence on the introduction of marginal analysis into economic theory. However, he contributed to generalizing critical microeconomics errors that reach our days. It can be said that almost all economists are formed with these errors. Jevons, Menger, and Walras perceived the problem with the classics' price theory and explicitly criticized them. Marshall's case is different. For this economist, the classical theory of prices was not wrong but incomplete and somehow imprecisely told. The theory of marginal utility served Marshall, as he claims, to perfect and not to refute the classics. Marshall used marginal utility theory to derive the demand curve for goods, but the supply curve is a cost curve.

Marshall (1890, p. 93 italics in original) explains the theory of diminishing marginal utility thus:

> There is an endless variety of wants, but there is a limit to each separate want. This familiar and fundamental tendency of human nature may be stated in the law of satiable wants or of diminishing utility thus: - The total utility of a thing to anyone (that is, the total pleasure or other benefit it yields him) increases with every increase in his stock of it, but not as fast as his stock increases. If his stock of it increases at a uniform rate the benefit derived from it increases at a diminishing rate. In other words, the additional benefit which a person derives from a given increase of his stock of a thing, diminishes with every increase in the stock that he already has.
> That part of the thing which he is only just induced to purchase may be called his marginal purchase, because he is on the margin of doubt whether it is worth his while to incur the outlay required to obtain it. And the utility of his marginal purchase may be called the marginal utility of the thing to him. Or, if instead of buying it, he makes the thing himself, then its marginal utility is the utility of that part which he thinks is only just worth his while to make. And thus the law just given may be worded: -
> The marginal utility of a thing to anyone diminishes with every increase in the amount of it he already has.

From here, Marshall begins to draw deductions until he reaches what he called the general law of demand and which he explains as follows (Marshall, 1890, p. 99 italics in original):

There is then one general law of demand: - The greater the amount to be sold, the smaller the amount must be the price at which it is offered in order that it may find purchasers; or, in other words, the amount demanded increases with a fall in price, and diminishes with a rise I price. There will not be any uniform relation between the fall in price and the increase of demand.

Marginal utility theory served Marshall to explain that larger available quantities of a given good are valued less and can therefore be sold at a lower price. In this way, he derived the decreasing demand curve, which is familiar in any economics text.

The Marshallian supply curve is not derived from marginal profits, as Menger did, but from costs: as the price of the product rises, the supply increases.

After assuming that "the normal supply price of any amount of that commodity may be taken to be its normal expense of production (including gross earnings of management)" (Marshall, 1890, pp. 342-343 italics in original), Marshall (1890, p. 345) performs the following analysis:

When therefore the amount produced (in a unit of time) is such that the demand price is greater than the supply price, then sellers receive more than is sufficient to make it worth their while to bring goods to market to that amount; and there is at work an active force tending to increase the amount brought forward for sale. On the other hand, when the amount produced is such that the demand price is less than the supply price, sellers receive less than
is sufficient to make it worth their while to bring goods to market on that scale; so that those who were just on the margin of doubt as to whether to go on producing are decided not to do so, and there is active force at work tending to diminish the amount brought forward for sale. When the demand price is equal to the supply price, the amount produced has no tendency either to be increased or to be diminished; it is in equilibrium.

The following quotes show how Marshall (1890, p. 347 italics in original) joins his conclusions to that of the classics. For Marshall, his findings did not invalidate the position of the classics but strengthened it:

These considerations point to the great importance of the element of time in relation to
demand and supply [...]. We shall gradually discover a great many different limitations of the
doctrine that the price at which a thing can be produced represents its real cost of production,
that is, the efforts and sacrifices which have been directly and indirectly devoted to its
production. For, in any age of rapid change such as this, the equilibrium of normal demand
and supply does not thus correspond to any distinct relation of a certain aggregate of
pleasures got from the consumption of the commodity and an aggregate of efforts and
sacrifices involved in producing it: the correspondence would not be exact, even if normal
earnings and interest were exact measures of the efforts and sacrifices for which they are the
money payments. This is the real drift of that much quoted, and much-misunderstood doctrine
of Adam Smith and other economists that the normal, or "natural," value of a commodity is
that which economic forces tend to bring about in the long run. It is the average value which
economic forces would bring about if the general conditions of life were stationary for a run
of time long enough to enable them all to work out their full effect.
Marshall is right to claim that the doctrine of Adam Smith and his followers was misinterpreted. For Smith, the short-term price, which he called the market price, was not influenced by production costs. Supply and demand could determine a market price below or above the natural or long-term price. As we saw in Part I, criticism of the classics for arguing that prices were determined by costs is very superficial. First, you must distinguish between market and natural prices; critics do not usually make this distinction. Therefore, Marshall's complaint is entirely fair. The classics were misunderstood. But just because they were misinterpreted does not mean their theory was correct. Classical theory has serious problems, but they are different from those pointed out by many of its critics. Marshall argued that the classics had an incomplete view, not an incorrect one, for having practically discarded the demand side and put all the emphasis on the supply or cost side. Marshall's (1890, pp. 348-349 italics in original) solution for price theory is given by the famous scissors paragraph:

We might reasonably dispute whether it is the upper or the under blade or a pair of scissors that cuts a piece of paper, as whether value is governed by utility or cost of production. It is the true that when one blade is held still, and the cutting is affected by moving the other, we may say with careless brevity that the cutting is done by the second; but the statement is not strictly accurate, and is to be excused only as long as it claims to be merely a popular and not a strict scientific account of what happens.
In the same way, when a thing already made has to be sold, the price which people will be willing to pay for it will be governed by their desire to have it, together with the amount they can afford to spend on it. Their desire to have it depends partly on the chance that, if they do not buy it, they will be able to get another thing like it at as low a price: this depends on the causes that govern the supply of it, and this again upon cost of production. But it may so happen that the stock be sold is practically fixed. This, for instance, is the case with a fish market, in which the value of fish for the day is governed almost exclusively by the stock on the slabs in relation to the demand: and if a person chooses to take the stock for granted, and say that the price is governed by demand, his brevity may perhaps be excused so long as does not claim strict accuracy. So again it may be pardonable, but it is not strictly accurate to say that the varying prices which the same rare book fetches, when sold and resold at Christie's auction room, are governed exclusively by demand.

Taking a case at the opposite extreme, we find some commodities which conform pretty close to the law of constant return; that is to say, their average cost of production will be very nearly the same whether they are produced in small quantities or in large. In such a case the normal level about which the market price fluctuates will be this definite and fixed (money) cost of production. If the demand happens to be great, the market price will rise for a time above the level; but as a results production will increase and the market price will fall: and conversely, if the demand falls for a time below its ordinary level.
In such a case, if a person chooses to neglect market fluctuations, and to take it for granted that there will anyhow be enough demand for the commodity to ensure that some of it, more or less, will find purchasers at a price equal to this cost of production, then he may be excused or ignoring the influence of demand, and speaking of (normal) price as governed by cost of production - provided only he does not claim scientific accuracy for the wording of this doctrine, and explains the influence of demand in its right place.
Thus we may conclude that, as a general rule, the shorter the period which we are considering, the greater must be the share of our attention which is given to the influence of demand on value; and the longer the period, the more important will be the influence of cost of production on value. For the influence of changes in cost of production takes as a rule a longer time to work itself out than does the influence of changes in demand.

This is a famous paragraph by Marshall in which he summarizes his thinking about the determinant of exchange value or price. In the short run, the price is determined by demand, in the long run by the costs of production, and in the medium run by the scissors, that is, by demand and supply, which is equivalent to marginal utility and production costs. For Marshall, as for traditional microeconomic theory, demand is determined by marginal utility and supply by marginal costs. As you can see, Marshall is saying the same thing as the classics noted but with the incorporation or foundation of marginal analysis.

Marshall departed quite a bit from the conclusions of Jevons, Walras, and especially Menger. These three economists argued, with all the imperfections you want to find in them, that the classical theory of prices was wrong. On the contrary, Marshall defends it. Appendix I of his book is called "Ricardo's Theory of Value;" in this appendix, Marshall (1890, p. 820) defends the theory of value (of change) of the classics, especially that of Ricardo:

Perhaps Jevon's antagonism to Ricardo and Mill would have been less if he had not himself fallen into the habit of speaking of relations which really exist only between demand price and value as though they held between utility and value; and if he had emphasized as Cournot had done, and as the use of mathematical forms might have been expected to lead him to do, that fundamental symmetry of the general relations in which demand and supply stand to value, which coexists with striking differences in the details of those relations. We must not indeed forget that, at the time at which he wrote, the demand side of the theory of value had been much neglected; and that he did excellent service by calling attention to it and developing it. There are few thinkers whose claims on our gratitude are as high and as various as those of Jevons: but that must not lead us to accept hastily his criticisms on his great predecessors.

Marshall came to the rescue of the classics; this is, in a way, fair, given that much of the criticisms leveled at them is wildly inaccurate. As we saw in Part $I$, the classics were misunderstood (especially Ricardo) since a careful reading clearly shows that they had given utility more importance than is believed in price determination. The classical price theory was inconsistent, and Marshall persisted in that inconsistency.

### 3.2. Vilfredo Pareto (1848-1923)

Pareto is a continuator of Walras within the so-called theory of general equilibrium but undoubtedly introduced tools of analysis of widespread use in the twentieth century. We can mention the "Paretian optimum," widely used in conventional microeconomics. Pareto, like Marshall, repeated the classical theory, but unlike Marshall, he does not seem to have noticed the overlap. To begin with, like almost all his
predecessors, he found problems with the terms value and utility due to the ambiguity of the words; he proposed a new term, ophelimite (Pareto, 1906, pp. 121-122):

It was mainly by the rectification of this error of the old economy that pure economics was born. With Jevons it appears as a rectification of the theories then underway on value, with Walras it becomes, and it was a great progress, the theory of a special case of economic equilibrium, that is to say that of free competition; while another case, that of monopoly, had already been studied, but in a very different way, by Cournot. Marshall, Edgeworth, Irving Fisher, have studied the economic phenomenon in a more extensive and more general way. In our course it became the general theory of economic equilibrium, and we go even further in this way in the present work. The word utility is led to mean, in political economy. anything other than what it may mean in everyday language. Thus, morphine is not useful, in the ordinary sense of the word, since it is harmful to the morphinomaniac, and on the contrary it is useful to him economically, since it satisfies one of his needs, even if it is unhealthy. Although the ancient economists have already mentioned this misunderstanding, it is still sometimes forgotten: it is also essential not to use the same word to indicate very different things. We have proposed in our course to designate economic utility with the word ophelimite, which other authors have adopted later. ${ }^{\text {ii }}$

We see that in the above quotation, Pareto did not manage to totally dispose of any "objective" influence of the scholastics. Valuation is always subjective, and there is no way to objectify it. If it could be objectified, we would assume that someone can determine good and bad. The differentiation between utility and ophelimity does not seem very justified and, in some ways, introduces more confusion.

Pareto's (1906, p. 129) confusion grows when using indifference curves (introduced by Edgeworth) as an analytical tool in consumer decision-making. This may have been one of the most critical errors incorporated into economic science.

> Take, for example, a man who lets himself be driven solely by his tastes and who owns 1 kilo of bread and 1 liter of wine, or vice versa. Consent, for example, to have 0.9 kg of bread in order to have 1.20 of wine. In other words, this means that these two combinations, namely 1 kilo of bread and 1 liter of wine. 0.9 of bread and 1.20 of wine are the same for him: he does not prefer the second to the first, nor the first to the second, he would not know which one to choose, and it is indifferent to him to enjoy one or the other of these combinations. Starting from this combination: a kilo of bread and a liter of wine, we find many others, among which the choice is indifferent [...].iii

The concept of indifference is self-contradictory. Economics is the science of choice, and choice is preference. Action implies that there can be no indifference. The use of mathematics led these economists down infertile paths. People choose based on an ordinal rather than a cardinal preference scale. In indifference curves, no matter how much you deny it, it is implicit that utility can be added and subtracted so that total utility remains constant. If Pareto had followed the path traced by the classical economists and the Austrians, he would not have fallen into this error. If choices are made based on a scale of ordinal preferences, then it becomes clear that there can be no indifference. People always opt for the goods or services that give them the most marginal utility.

On the other hand, the indifference curves are limited to the case of substitute goods. One might think about eating fewer red and white meats or swapping one drink for another. But how can you compensate for the loss of usefulness of stopping eating by watching more TV or, worse, swimming more? An analysis based on preference scales prevents falling into this type of inaccuracy. Generalization is crucial for any theory, but there is a significant difference between generalization and losing track of what needs to be explained.

Based on this error, but with a supposed mathematical rigor, Pareto (1906, pp. 267-268) continues with his analysis arriving at the property of equilibrium, today is known as Pareto optimal:

The balance, according to the conditions in which it is obtained, enjoys certain properties that are important to know.


#### Abstract

We will begin by defining a term that is good to use to avoid extending too much. We will say that the members of a collectivity enjoy, in a certain position, the maximum of ophelimite, when it is impossible to find a means of moving very little away from that position, so that the ophelimite enjoyed by each of the individuals of this collectivity increases or decreases. That is to say that any small displacement from this position necessarily has the effect of increasing the ophelimite enjoyed by certain individuals and diminishing that enjoyed by others; of being pleasant to some and unpleasant to others. ${ }^{\text {iv }}$


The curves took Pareto down dubious paths. He believed that these curves could be calculated in practice in the same way he drew them in his book (Pareto, 1906, pp. 264-275):

The collectivist state, better than free competition, seems to be able to bring the equilibrium point along the lines of complete transformations. Indeed, it is difficult for a private company to follow exactly in its sales the line of complete transformations. He must therefore make himself pay for his customers, in principle the general expenses, and then sell them the goods at cost price, deducting the general expenses. Except in particular cases it is not clear how this could happen. The socialist state, on the contrary, can impose on the consumers of its commodities the general costs of producing these commodities, and then transfer them at cost price. You can, therefore, follow the line of complete transformations. ${ }^{v}$

Pareto was not a socialist. He also saw deficiencies and virtues in the capitalist system; Pareto (1906, p. 275) concludes:

In short, pure economics does not give us a truly decisive criterion for choosing between an organization of society based on private property and a socialist organization. This problem can only be solved by considering the other characters of the phenomena. ${ }^{\text {vi }}$

Pareto could not get to the essence of pricing. If he had done so, he would have understood that pure economics shows that a socialist economy cannot be efficient and that a capitalist economy is always the most efficient system possible. However, it is obviously not efficient in the Paretian sense. In the conclusions, we will analyze this point.

We have seen that for all his systems of simultaneous equations, Walras even had a brief paragraph in which he criticized the classics. Walras explicitly stated that costs did not determine prices, as the classics argued, but was precisely the other way around. Marshall is, in this sense, a step backward from Jevons and Walras. Pareto (1906, p. 275) also is a step back:

The cost of production has been conceived by literary economists as a normal price around which prices determined by supply and demand should gravitate, and thus they have come to take into account, albeit in a very imperfect form, three categories of conditions [...]. But they consider them independent of each other, it seems that the cost of production of a commodity was independent of the prices of this commodity and of the others. It's easy to see how rude the mistake is. For example, the cost of producing coal depends on the price of machines, and the cost of producing machines depends on the price of coal. Consequently, the cost of producing coal depends on the price of that same coal. And it depends even more directly on whether the coal consumption of the machines used in the mine is considered.
The price or exchange value is determined at the same time as the economic equilibrium, and that which arises from the opposition between tastes and obstacles. He who looks only one way and considers only tastes, believes that these exclusively determine price, and finds the cause of value in utility (ophelimite). He who looks at the other side, and considers only the obstacles, believes that these are exclusively those that determine the price and finds the cause of value in the cost of production. And if among the obstacles only work finds the cause of value exclusively in work. If in the system of conditions (equations) which, we have already seen, determine equilibrium, we suppose that all conditions are satisfied in themselves, with the exception of those relating to labor, we may say that value (price) depends only on labor,
and that this theory will not be false, but simply incomplete. It will be true as long as the hypotheses made are realized.

Pareto's conclusion is the same as Marshall's scissors. There is no teleological relationship between prices and costs but a simultaneous determination. It is not just costs or utility that determine prices. It is the interaction of both that defines them. Looks like Pareto did not read Walras carefully. Otherwise, he must have explained why Walras firmly maintained that classical theory was erroneous and not incomplete, as Marshall and Pareto claim. In any case, Pareto and most economists were carried away by the system of Walrasian equations, where a simultaneous determination of prices by utility and costs is obviously implicit. In Walras, there is a contradiction between his equations and the paragraph against the theory of the classics. In Pareto's case, there is, in principle, no contradiction, only error.

Finally, it is worth remembering that Pareto, like Walras, had a poor mathematical background. Pareto did not realize Walras's mistake of assuming that the system must have a unique solution because there are as many equations as there are unknowns. If these economists had known more about mathematics and economics, economic theory would have followed more fertile paths than those in the twentieth century.

### 3.3. Gustav Cassel (1866-1945)

With Cassel, the reversion of the theory of value and price takes place in great strides. Cassel seems to have jettisoned all the progress made so far. The use of mathematical tools blinded him more than Pareto. Let's look at the following quote (Cassel, 1932, p. 49):

Consequently, it follows that a special theory of value is, to say the least, quite unnecessary in economics. Every attempt to frame such a theory without using a common measure to express estimates of value must encounter great difficulties. But as soon as such a common measure is introduced, money in its essence is postulated. Values are then replaced by prices, valuation by pricing, and we have a theory of prices instead of a theory of value. From this we must conclude that the whole of the so-called theory of value ought to be discarded in economics. The theoretical exposition of the exchange economy must, from the start, take money into consideration, and thus be essentially a theory of pricing.

With this paragraph, Cassel seems not to have understood the very essence of economics: the choice between scarce means. While valuations determine prices, value and price are distinct concepts. Currency has nothing to do with either value or price determination. Money emerges as a necessity in the face of the growing difficulties that barter generates when labor and trade divide.

Cassel's (1932, pp. 145-146) conclusion is clear:
There has been a great deal of discussion as to what are the factors determining prices. This question can now be answered. The determining factors of price are the different given coefficients of our equations. These coefficients may be classified in two main groups, which we may call the objective and subjective factors determining price. The objective factors are partly the quantities of the factors of production $(R)$, and partly the so-called technical coefficients ( $a$ ). The subjective factors are the coefficients of equations (4), which show the dependence of demand upon prices. All these factors are essential in determining prices. An "objective" or "subjective" theory of value, in the sense of a theory that would attribute the settlement of prices to objective or subjective factors alone, is therefore absurd; and the whole of the controversy between these theories of value, which has occupied such a disproportionately large place in economic literature, is a pure waste of energy.

As you can see, Cassel goes much further than Marshall and Pareto in the theory of value. For him, the discussion is a waste of time. Waste of time or not, Cassel's theory ends up being inferior to the classics. Their conclusions are the same: In the long run, prices and costs are equalized. The simultaneous equations did not allow these economists to see that all action involves time. Production takes place before price; therefore, costs are before prices. Using productive factors (with their technical coefficients of production) precedes price.

Consequently, it is not clear how production costs or technical factors can have anything to do with price formation. As Menger did, time forces us to distinguish between prices and expected prices. Production costs interact, at best, with expected prices. When determining the price of a good or service, costs are history. Although the producer will try to recover the costs, only the subjective valuation determines how much will be paid for the good or service. Cassel lost all economic compasses. It is yet another example of what the "rigor" of mathematical tools can do.

### 3.4. Eugene von Böhm-Bawerk (1851-1914)

Böhm-Bawerk is, along with Marshall, the economist who most influenced the diffusion and acceptance of marginal utility theory. His exposition was so clear and systematic that he defeated any attempt to maintain the reputation of the classics at this point. He also made noticeably clear the differences between the economists of the Austrian school and the rest of the marginal islands. To some extent, however, he regressed from Menger in explaining the price of productive factors as the present value of the final goods produced. Menger was more precise since the price of productive factors is determined by the expected present value of the final goods to be produced. In any case, his analytical exposition was very forceful. Böhm-Bawerk (1891, pp. 135-136, italics added) starts by defining value as follows:
[...] we shall define it, unambiguously and exactly, as: That importance which goods or complexes of good acquire, as the recognized condition of a utility which makes for the wellbeing of a subject, and would not be obtained without them.
All goods have usefulness, but all goods have not value. For the emergence of value there must be scarcity as well as usefulness - not absolute scarcity, but scarcity relative to the demand for the particular class of goods. To put it more exactly: goods acquire value when the whole available stock of them is not sufficient to cover the wants depending on them for satisfaction, or when the stock would not be sufficient without these particular goods. On the other hand, those goods remain valueless which are offered in such superfluity that all the wants which they are fitted to satisfy are completely supplied, and when, beyond that, there is a surplus which can find no further employment in the satisfaction of want, and which, at the same time, is large enough to spare the goods or quantities of goods that we are valuing without imperiling the satisfaction of any one want.

As we can see, value arises from the conjunction of utility and scarcity. A good that is not useful has no value, and a good that is not scarce also has no value, no matter how useful it may be. In addition, Böhm-Bawerk made it clear that it arises from the perceived usefulness of people. A good does not acquire utility until people perceive that a thing can be used to satisfy some of their needs. Still, even when that utility is perceived, it must be scarce for the good to acquire value. Scarcity is a relative concept; it does not refer to the "objective" quantity available but to the amount available in relation to what is needed. A liter of water can be very abundant if we are not thirsty and very scarce in the middle of a desert.

Böhm-Bawerk (1891, pp. 148-149) explains that the last satisfied need determines the value of goods in the following way:
[...] the value of a good is measured by the importance of that concrete want, or partial want, which is least urgent among the wants that are met from the available stock of similar goods. What determines the value of a good, then, is not its greatest utility, not its average utility, but the least utility which it, or one like it, might be reasonably employed in providing under the concrete economical conditions. [...] we shall follow Wieser in calling this least utility - the utility that stands on the margin of the economically permissible - the economic Marginal Utility of the good. The law which governs amount of value, then, may be put in the following very simple formula: The value of a good is determined by the amount of its Marginal Utility. This proposition is the key-stone of our theory of value. But it is more. In my opinion it is the master-key to the action of practical economic men with regard to goods. In the simplest case, as in all the tangle and complication which our present varied economic life has created, we find men valuing the goods with which they have to deal by the marginal utility of these goods,
and dealing with them according to the result of this valuation. And to this extent the doctrine of marginal utility is not only the key-stone of the theory of value, but, as affording the explanation of all economical transactions, it is the key-stone of all economical theory.

From the point of view of the history of economic thought, we can say with Schumpeter that Böhm-Bawerk understood the importance of the theory of marginal utility. In other words, for Schumpeter, it is of little use for someone to say something if they do not realize the significance of what they are saying. Theoretical advances and revolutions occur when the one who affirms something recognizes its importance and consequences. In the case of Böhm-Bawerk, this realization of the significance of marginal utility theory is apparent, as seen in the previous quote. With the following example Böhm-Bawerk (1891, pp. 149-151) clarifies the concept of marginal utility:

A colonial Farmer, whose log-hut stands by itself in the primeval forest, far away from the busy haunts of men, has just harvested five sacks of corn. These must serve him till the next autumn. Being a thrifty soul he lays his plans for the employment of these sacks over the year. One sack he absolutely requires for the sustenance of his life till the next harvest. A second he requires to supplement his bare living to the extent of keeping himself hale and vigorous. More corn than this, in the shape of bread and farinaceous food generally, he has no desire for. On the other hand, it would be very desirable to have some animal food, and he sets aside, therefore, a third sack to feed poultry. A fourth sack he destines for the making of coarse spirits. Suppose, now, that he various personal wants have been fully provided for by this apportionment of the four sacks, and that he cannot think of anything better to do with the fifth sack than feed a number of parrots, whose antics amuse him. Naturally these various methods of employing the corn are not equal in importance. If, to express this shortly in figures, we make out a scale of ten degrees of importance, our farmer will, naturally, give the highest figure 10 to the sustenance of his life; to the maintenance of his health he will give, say, the figure 8 ; then, going down the scale, he might give the figure 6 to the improvement of his fare by the addition of meat, the figure 4 to the enjoyment he gets from the liquor, and, finally, to the keeping of parrots, as expressing the least degree of importance, he will give the lowest possible figure 1. And now, putting ourselves in imagination at the standpoint of the farmer, we ask, What in this circumstance will be the importance, as regards his wellbeing of one sack of corn?
This, as we know, will be most simply tested by inquiring, How much utility will be lose if a sack of corn gets lost? Suppose we carry out this in detail. Evidently our farmer would not be very wise if he thought of deducting the lost sack from his own consumption, and imperiled his health and life while using the corn as before to make brandy and feed parrots. On consideration we must see that only one course is conceivable: with the four sacks that remain our farmer will provide for the four most urgent groups of wants, and give up only the satisfaction of the last and least important, the marginal utility - in this case, the keeping of parrots. The only difference, then, that his having or not having the fifth sack of corn makes to his wellbeing is that, in the one case, he may allow himself the pleasure of keeping parrots, in the other he may not; and he will rightly value a single sack of his stock according to this unimportant utility. And not only one sack, but every single sack; for, if the sacks are equal to one another, it will be all the same to our farmer whether he lose sack A or sack B, so long as, behind the one lost, there are still four other sacks for the satisfying of his more urgent wants. To vary the illustration, assume that our farmer's wants remain the same, and that he has only three sacks of grain. What now is the value of one sack to him? The test again is quite easily applied. If he has three sacks he can and will provide for the three most important groups of wants. If he has only two sacks he will be obliged to limit himself to the satisfying of the two most important groups and give up the satisfying of the third, that of animal food. The possession of the third sack - and the third sack, be it remembered, is not a definite sack but any of the three sacks, so long as there are other two behind it - directly carries with it, therefore, the satisfaction of his third most important want; that is, the last or least of those wants covered by the three sacks which constitute his total stock. Any estimate other than that
according to the marginal utility would, in this case also, obviously run counter to facts, and would be quite incorrect.
Finally, suppose that our farmer's wants remain as before, and that he only possesses one single sack of corn. In this case it is perfectly clear that all less important methods of employing the corn are out of court, and that it will be devoted to and spent in sustaining the farmer's lie - a function for which it just suffices. And it is as clear that if this single sack fails the farmer will no longer be able to support himself in life. His possession of the sack, therefore, means life; his loss of it means death; the single sack of corn has the greatest conceivable importance for the wellbeing of the farmer. And all this is still in conformity with our principle of marginal utility. The greatest utility - the preservation of life - is here the sole, as well as the last or marginal utility.

As in Menger's case, for Böhm-Bawerk, price is determined solely by the interaction of people's marginal utilities. Price does not measure or coincide with marginal profits. The role of marginal utilities is to set upper and lower limits on prices that match supply and demand. The marginal utility determines how much a person is willing to pay for a good. She will make the purchase if the marginal utility of what she buys is greater than that of what she must give up, but in no way does it "measure" the price. We can develop an example by BöhmBawerk (see 1891, p. 203):

Table 1. Price determined solely by marginal utility

| Buyer | Buyer's value of a horse | Seller | Seller's value of a horse |
| :---: | :---: | :---: | :---: |
| Aa | 300 | Ba | 100 |
| Ab | 280 | Bb | 110 |
| Ac | 260 | Bc | 150 |
| Ad | 240 | Bd | 170 |
| Ae | 220 | Be | 200 |
| Af | 210 | Bf | 215 |
| Ag | 200 | Bg | 250 |
| Ah | 180 | Bh | 260 |
| Ai | 170 |  |  |
| Aj | 150 |  |  |

In this example, the price must be above $\$ 210$ and below $\$ 215(\$ 210<p<\$ 215)$ to match the quantity supplied and the quantity demanded. If the price is $\$ 210$, the quantity demanded will be 6 horses, and the amount offered will be 5 . On the contrary, if the price is $\$ 215$, the quantity demanded will be 5 horses and the one offered, 6 . Therefore, the price that equals the amount demanded and supplied can be anything between $\$ 210<p<\$ 215$. However, within these limits, the buyer $A f$ does not buy, and the seller $B f$ does not sell. Yet, they put the boundaries "with their marginal profits" at the market price. The marginal buyer and seller are $A c$ and $B e$, respectively. As you can see, marginal utility sets the limits within which the equilibrium price can fluctuate. Still, as mathematical economists concluded, it is not true that prices equal marginal utility. This only happens by introducing convenient assumptions to use mathematical tools (mainly calculus). Still, it obviously leads to less rigorous conclusions.

For Böhm-Bawerk (1891, p. 224 italics added), as for Menger, production costs do not influence determining costs. On the contrary, costs are a consequence of prices:

The formation of value and Price takes its start from the subjective valuations put upon finished products by their consumers. These valuations determine the demand for those products. As supply, over against its demand, stand, in the first instance, the stocks of finished commodities held by producers. The point of intersection of the two-sided valuations, the valuation of the
marginal pairs, determines, as we know, the price, and, of course, determines the price of each kind of product separately.

As we can see, Böhm-Bawerk eliminates production costs from pricing. The production process is one thing, and the formation of prices is another. The supply curve constructed based on marginal costs ignores this significant difference. At the time of selling, the costs are history. For the seller, the price will be determined by the buyers' valuation (based on marginal utility) and by the valuation that the seller makes of his available stock. Obviously, the seller will want to recover their costs, but "wanting" is distinct from "power."

However, Böhm-Bawerk took a step back from Menger by wanting to explain that costs depend on prices and not prices on costs, as the classics argued. Menger argued that the productive factors' prices depend on the final goods' expected prices. Instead, Böhm-Bawerk argued that they relied on prices, which was a mistake. When deciding on production, obviously, the goods are not available; they are simply a project. So, the prices that the producer is willing to pay for the productive factors depend on the expected price of the good he wants to produce. Böhm-Bawerk (1891, p. 226 italics added) makes his explanation in this way:

And now we have to consider the causal connection which has ended in this price. It runs, in the clearest possible way, in an unbroken chain from value and price of products to value and price of costs - from iron wares to raw iron, and not conversely. The links in the chain are these. The valuation which consumers subjectively put upon iron products forms the first link. This helps, next, to determine the figures of the valuation - the money price at which consumers can take part on the demand for iron products. These, prices, then, determine, in methods with which we are no familiar, the resultant price of iron products in the market for such products. This resultant price, again, indicates to the producers the (exchange) valuation which they in turn may attach to the productive material iron, and thus the figure at which they may enter the market as buyers of iron. From their figured, finally, results the market price of iron.

Finally, he adds (p. 227):
It is not this cost Good, then, that dictates its fixed Price to the products that proceed from it; on the contrary, it receives its own price by the medium of the price of his products [...]

Böhm-Bawerk was clear enough to show the classics and Marx's errors in their price theory. The logic of his analyses was solid. But it would have been much more precise if, following Menger, he had distinguished more precisely between price and expected price. After introducing his theory of interest, he goes further in his analysis, concluding that the price of productive factors is determined by the "present" value of final goods. It would have been much better to say the expected present expected value of the final goods.

Böhm-Bawerk and Wieser attached much importance to the problem of value, much more so than their marginalist colleagues. In the case of Böhm-Bawerk, there are three essential writings: (1) "The Austrian Economists," ${ }^{12}$ (2) "Unresolved Contradiction in the Marxian Economic System," ${ }^{13}$ and (3) "The Ultimate Standard of Value."14 In these articles, Böhm-Bawerk continued to expand his analysis of value to show the error of the theory of the classics and also marked the differences between the theory of the Austrian school and those of the other marginal island schools. Referring to the theory of marginal utility, Böhm-Bawerk (1962, p. 10) says:

As is well known, the fundamental principle of this theory of the Austrian school is shared by certain other economists. A German economist, Gossen, had enunciated it in a book of his which appeared in 1854, but at that time it attracted not the slightest attention. Somewhat later the same principle was almost simultaneously discovered in three different countries, by

[^8]three economists who knew nothing of one another and nothing of Gossen - by the Englishman W.S. Jevons, by C. Menger, the founder of the Austrian school, and by the Swiss Walras. Professor J.B. Clark, too, an American investigator, came very near the same idea. But the direction in which I believe the Austrians have outstripped their rivals, is the use they have made of the fundamental idea in the subsequent construction of economic theory.

Comparing Jevons and Walras with Austrian economists to the solution they gave to the "vicious circle" of the classics, Böhm-Bawerk (1962, p. 10) argues:

It is well known that Jevons and Walras arrived at a similar law of price. Their statement, however, has considerable deficiencies, which were first supplied by the Austrians. It was the latter who first found the right way of escape from the circulus vitiosus in which the older theory of price as dependent upon supply and demand was involved.

In his article "The Ultimate Standard of Value," Böhm-Bawerk (1894, pp. 8-9) points out that there are economists, such as Jevons, who explained value exclusively on the basis of marginal utility while others, such as Scharling, insisted on defending a theory of cost. In the middle are eclectic positions like Marshall's and Edgeworth's, which are put halfway:

> Where opinions vary so widely from one another, some one is usually found who will take middle course, hoping to find a solution for the problem in the golden mean. This mission of conciliation has been undertaken in this case by no less eminent economists than Professor Marshall, of Cambridge, and Professor Edgeworth, of Oxford. Both of these writers incline toward the theory of marginal utility, but have perched themselves very nicely upon the middle round of the ladder, from which vantage-ground they send forth gentle blame and conciliating applause to both parties in the discussion. Jevons and the Austrian economists are censured for exaggerating the importance of marginal utility, while the adherents of the classical theory are taken to task for underrating its importance; the truth, they say, lies in the middle. Scarcity goods, without doubt, have their value determined entirely by utility, and the supply by cost; since the price is determined by the interaction of these two factors, one cannot say either that utility or that cost alone determines value; but rather that utility and cost cooperate with each other in the determination of price, like, to use Professor Marshall's figure, the two blades of a pair of shears.

Böhm-Bawerk develops a lengthy critique of the Marshallian theory of prices (utility and costs), which is also, as we saw earlier, that of Pareto, Cassel, and Walras to a significant extent. Although Böhm-Bawerk was not as precise as Menger in abandoning price expectations, he managed to clear out the errors of the theory of classics. Böhm-Bawerk did not perfect or complete the theory of the classics but, like Menger, gave it a turn of one hundred and eighty degrees. In Marshall's case, we saw that he ended up with a defense of the theory of costs of production. Marshall used marginal utility theory to "perfect" or "complete" classical theory. Menger and Böhm-Bawerk used marginal utility theory to show that classical price theory was wrong. This is a significant difference between the economists of the Austrian school and the marginalist schools of England and Switzerland.

Our next and last step is to see how marginal price theory developed from these first marginalist steps.

## 4. The Current State of the Theory of Value and Price

Marshall's influence was decisive in the theory of value and price during the twentieth century. Somehow Cassel's comment that "[...] A theory of value, objective or subjective, that merely refers prices to objective or subjective determining causes is meaningless, and the whole struggle between these theories of value, which occupies so much space in the economic literature, is wasted time," means a significant setback. Indeed, conventional microeconomic theory, with which almost all the world's economists are trained today, does not mention the value problem. And its further development is essentially Marshallian or, as Böhm-Bawerk said, eclectic. According to these books, prices are determined by a subjective factor, marginal utility, which
determines the demand curve, and by an objective factor, marginal costs, which determines the supply curve. Without making a serious mistake, the marginalists could be divided into two groups: 1) the mathematicians and 2) the Austrians or literary. The former made such several critical mistakes that it can be said that the classics had a more solid economic theory despite their flawed approach to prices. Mathematical marginalists have worsened the position of economic theory relative to the classics. Not only because of the model's assumptions but also because of logical inconsistencies even when accepting the assumptions.

Mathematical marginalists created the fiction of perfect competition. This is an ideal world with infinite consumers and producers, so neither can affect the market price by increasing or decreasing their demand or supply. All economic agents have perfect knowledge, so there can only be a single price for the same product. The goods are entirely homogeneous; there is not the slightest difference between them; the slightest difference makes the products different. The productive factors are perfectly substitutable and divisible. ${ }^{15}$

Under these assumptions, it has been concluded that a Pareto optimum is reached if there is perfect competition. The allocation of productive resources is efficient; no one's situation can be improved without harming others. Two books published in 1933, Joan Robinson's The Economics of Imperfect Competition and E. H. Chamberlin's The Theory of Monopolistic Competition, attempt to give the theory greater realism and had significant influence. After all, the world did not respond to the assumptions of perfect competition. In any case, these authors followed infertile paths and helped to propagate the errors.

It is generally accepted today that assumptions are unrealistic, but, as in physics, models are explanatory simplifications of reality. ${ }^{16}$ Therefore, unrealistic assumptions can be fertile if theory allows good predictions. However, in the remainder of this work, I want to show that this evolution of mathematical marginalist theory has led to fundamental errors that we can group into two classes: (1) logical errors that invalidate the theoretical conclusions, especially concerning the concept of economic efficiency, and (2) errors that follow from assumptions that result in less fertile theories intended to explain facts. After all, the goal of any theory is to make it possible to explain reality.

A fatal logical error. Conventional microeconomic theory concludes that in perfect competition, optimum efficiency is achieved. In perfect competition, the demand curve of each firm is perfectly elastic. Therefore, the price of the product is equal to marginal revenue. Given the model's assumptions, the perfectly competitive firm maximizes profits when the price (or marginal revenue) equals marginal cost. On the other hand, in imperfect competition, each producer has some influence on the price (the extreme case being the monopolist facing the total demand curve of the market). Therefore, if there is imperfect competition, the demand curves faced by firms have a negative slope; they are no longer perfectly elastic. As a result, marginal revenue no longer coincides with the price; it falls more rapidly. Under these conditions, the firm maximizes profits in the volume of production that equals marginal revenue and marginal cost. This implies that in imperfect competition, a smaller quantity is produced and sold at a higher price than would be produced and sold in a market of perfect competition. The two graphs show the two extreme cases: (a) that of perfect competition and (b) that of monopoly. According to traditional microeconomics, a monopoly firm produces the quantity where marginal revenue equals marginal cost because, at this point, profits are maximized. This takes place at point $B$ of the imperfect competition figure. If the market were perfect competition, it would occur at point $A$ of the imperfect competition figure. As you can see, at point $B$ the production is lower than at point $A$, and the price is higher. The conclusion is that resource allocation is not optimized. A system of perfect competition would produce a greater quantity at a lower price.

[^9]Figure 1. Perfect vs. imperfect competition

## Perfect competition



This analysis, so prevalent in all microeconomics books, contains a mathematical error that invalidates all conclusions. The error can be explained as follows. The theory states that the demand curve directed to each firm is perfectly elastic, so the price is constant. But at the same time, it claims that the demand curve of the entire market has a negative slope. Making both claims is inconsistent. If the total market demand curve has a negative slope, then the individual demand curves must have an infinitesimal slope. The monopolist faces the whole demand curve of the market. As more producers are added, individual demand curves become more elastic. If producers tend to be infinite, as the perfect competition model assumes, the slope of individual curves tends to zero but could never be zero. These individual curves have an infinitesimal slope. Mathematical economists, who do not seem to know mathematics very well, take a small leap by claiming that the curve can be considered perfectly elastic. ${ }^{17}$ They have not realized what a big mistake they have made. Suppose the individual demand curves have an infinitesimal slope. In that case, it ceases to be true that the price is equal to marginal revenue. There must be an infinitesimal difference between both curves. Therefore, the competitive

[^10]firm will not maximize profits where the price equals the marginal cost. This producer, like the monopolist, maximizes profits where marginal revenue equals marginal cost. He will produce infinitesimally less and sell at an infinitesimally higher price. It follows that if every producer's output equals marginal revenue with marginal cost, then at the aggregate level, they will all produce at the same point a monopolist would produce (if the cost structures coincide with those of the monopolist). Thus, we can conclude that a model of perfect competition is "not" necessarily more efficient than a monopoly market.

Logical consistency makes it contradictory to say that while individual demand curves are perfectly elastic, the total demand curve has a negative slope. If the individual demand curves are perfectly elastic, the entire demand curve must also be perfectly elastic. But in this case, the monopolist would also face a horizontal demand, and there would be no difference between perfect competition and a monopoly. If the total demand curve has a negative slope, the individual demand curves must also have a negative slope. But in this case, it is no longer true that the price equals marginal income. Some economists argue that the individual demand curve is the one that the producer perceives. Still, since the model assumes perfect information, this line of argument is inconsistent. Given this assumption, what the producer perceives cannot be different than reality. If their perception differs from reality, microeconomics theorists should be more rigorous in explaining what they mean by perfect information. On the other hand, if in a market of perfect competition, nobody can move the price, who moves the price? It is like saying that many men with zero force come together to move a great stone. In this case, the stone would never move. But if each man exerts a little force, all together can move it. Professors K. J. Arrow and F. H. Hahn (1971, p. 325) noted the inconsistency of the model:

Thus, for instance, in a production economy, if every firm faces a horizontal demand curve (or thinks that it faces such a curve), it is not easy to visualize any firm changing the price at which its product is sold. What is happening now is that, having decided on one idealization (perfect competition), we run into what must be taken to be logical difficulties unless we import a further idealization: the auctioneer.

The mistake is so gross that one could say that mathematical economics is neither economics nor mathematics.

Due to this error, all the essential conclusions that we can list as follows collapse:

1. A monopolist is not necessarily inefficient. A monopoly is more efficient if it operates at lower costs than many aggregate producers. Economists of the Austrian school distinguished between legal and market monopolies. A legal monopoly arises from a law that prevents competition. This monopoly is "necessarily" ineffective. A monopoly displaces the rest of the competition because it is the most efficient. As Mises (1922, p. 344) noted, "No other part of economic theory has been so misunderstood as monopoly theory."
2. Competition should not be associated with the "quantity" of producers operating in the market but with freedom of entry and exit to the market, i.e., the absence of legal obstacles. Without legal barriers, the market is competitive if potential competition is possible.
3. The definition of efficiency in traditional microeconomic theory is logically inconsistent. The dominant dogma that a market of perfect competition achieves an optimum is supported by a substantial mathematical error. This does not seem to leave rigorous mathematical economics very well. Mathematical economists seem to have come to more inconsistent conclusions.

The errors of the assumptions. No one disputes that a theory is a simplification of reality. We would gain nothing by trying to reproduce reality as it is. As mentioned, a 1:1 scale map is of no use. In any case, it is one thing to simplify reality and another to distort it or draw conclusions that do not allow for an explanation.

The model of perfect competition, associated with the economic optimum, assumes perfect information on the part of economic agents. This simplification of reality has not allowed mathematical economists to clearly distinguish between price and expected price. As is known, all production takes time. Taking time away from economic theory rather than simplifying reality distorts it. The Austrian school assumed information, rather than the market, is atomized. ${ }^{18}$
${ }^{18}$ See Mises (1949, Chapter VI), Hayek (1948, Chapter IV, V, 1978, Chapter 12), and Kirzner (1973).

Business decisions to produce and invest are made based on expected prices. Entrepreneurs produce because they believe they will profit and produce where they expect earnings. As we saw in Menger, the prices they are willing to pay for productive factors depend on the expected prices of the goods that these productive factors will produce. Production costs are higher than prices. Once production is finished, costs are history, and they can no longer be modified. Now the entrepreneur has a stock of goods and will try to sell them at the highest possible price. But trying is not the same as power. The final good price is determined by the marginal utility of the buyers and sellers. Production costs have no influence on the determination of prices. If the producer observes that he does not get a price that covers his costs, he may be reluctant to sell at first. Unless buyers' valuation (marginal utility) changes, they will have to resign themselves to keep the stock. What you decide to do depends on your own marginal utility.

The final decision depends largely on the type of good. Suppose the good is quickly perishable, such as bread. In that case, the producer will likely have to resign to lower the price faster. Otherwise, the more time passes, the less you get for the product. Another example may be the case of computers. They become obsolete as time passes, and their price drops irretrievably. If the good is not quickly perishable, the producer may decide to wait to see if conditions change in his favor.

There may also be other factors that make the producer rush to sell, for example, the illiquidity caused by the accumulation of stocks.

In any case, past prices and costs will encourage or discourage future production. But if it is about being rigorous, the entrepreneur projects present prices into the future. Obviously, much of the prices will likely remain unchanged in the short term, say the following hours, days, and maybe months. But as we look further into the future, it becomes increasingly clear that expected prices count for decision-making, not prices. Costs have no bearing on pricing. Perhaps the only impact they may have is that the producer is reluctant to sell if the price does not allow them to be recovered.

The eclectic solution, as Böhm-Bawerk called Marshall and the general equilibrium's approach, is as flawed as classic theory. But one might add that the mathematical school meant a critical step backward for economic science. Classical economists had been wrong to conclude that costs determine prices. Still, they could explain the market process more accurately than modern microeconomics. The classics show why a free market achieves the best allocation of productive resources. Mathematical economics ventured down a path in which nothing could be proven. If we use the fertility of theory as a parameter to explain how the economic world works, the error of the classics was less important than the logical inconsistency of mathematical economics to define an optimum.

The effects of any human decision, whether economic or not, can be divided into ex-ante effects and ex-post effects. Every decision is determined by the ex-ante effects that the person believes will occur. The ex-post effects are the real ones. For example, the evaluation of an investment project results in ex-ante effects. If the project is carried out, we can observe the ex-post results. The problem is that there is no objective way to determine whether a decision is good or bad. The only way to know if a decision is good or bad is by its ex-post results. It is not within human capacity to predict the future with certainty. Different methods, mathematical or not, can be used to help make predictions, but all of them are subjective.

It can be concluded that every decision is necessarily efficient. The one who makes the decision, does so thinking it is the best course of action. You may or may not agree with the decision, but it is impossible to prove objectively that it is right or wrong.

Suppose an economic theory is constructed from atomized knowledge. In that case, one arrives at different results from those by assuming perfect knowledge. You have to change the definition of optimal since there will always be errors. Uncertainty forces us to show how the market works as a process. When decisions are made under conditions of uncertainty, three things can happen: (1) prices have been accurately estimated; in this case, the ex-ante results and ex-post results coincide; (2) ex-ante prices are above ex-post prices, and (3) exante prices are below ex-post prices. In the first case, you have a situation of equilibrium and the last two of imbalance: one where the market is overestimated and another where it is underestimated. Economists of the Austrian school emphasize explaining how the market adjusts. If everyone foresaw precisely, we would be within the assumptions of perfect knowledge. But the reality is that there are continuous errors in one direction or another that require a process of adjustment.

Economic theory needs to redefine the concepts of efficiency and optimal if it wants to explain the real world. It can be said that the optimum is found when the errors of economic agents are minimized. This is what the Austrian school's economists sought; for this reason, they achieved a much more fertile theory.

The mathematical branch of marginalism has assumed perfect knowledge in its models; In this way, there can be no difference between expected prices and prices. The expected prices have to match the prices that are formed. Or the ex-ante effects of any decision must coincide with the ex-post effects. Perhaps the lack of uncertainty in these models did not allow to see with the same clarity as the Austrians the informative function of prices. And for this reason, they demonstrated that it is impossible for a socialist society, without private property, to be efficient. And that a free enterprise system is the only one that can minimize errors, but obviously, they can never be eliminated.

## 5. Summary

The theory of value and price was born within the field of ethics. The first philosophers tried to answer justice problems and wondered what the just price was. In this early stage, philosophers did not ask themselves what a price was or how it was determined and fluctuated. This doctrine was born with Aristotle and was continued by the Scholastics for many years. However, to determine the fair price, the scholastics began to introduce more analysis. They slowly began to explain what a price was and how it was determined, with many inaccuracies but making great strides. In particular, the school of Salamanca made significant analytical advances. In their analyses, subjective, objective, and ethical concepts were mixed, but they were nevertheless an important advance for economic science.

After the ethical stage, we move on to pure theory, where considerations of justice remain in the background. Classical economists are the ones who gave birth to this stage of theoretical and systematic analysis of economic science. But they made the mistake of concluding that costs determined prices and fell into a vicious circle from which they could not get out.

The marginalists tried to find a solution to the problem, but not all of them succeeded. The English and Swiss marginalists ended up with an eclectic solution in which prices were not determined only by costs as the classics held and were not determined only by marginal utility. For them, prices were determined by a subjective factor (marginal utility) and an objective factor (costs). For their part, the Austrian marginalists turned 180 degrees with respect to the classics, concluding that marginal utility is the only determinant of prices.

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Juan C. Cachanosky

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${ }^{\mathrm{i}} \mathrm{TN}$ : This article was originally published in 1995, in LIBERTAS, volume 22 (May).
The translation includes some grammatical revisions for readability and clarity. The original meaning and emphasis of the text has been preserved striving to be loyal to the original text in Spanish.
I appreciate the contribution of Peter Lewin to this translation. Any misrepresentation from the original text is my own doing.
ii TN: My translation. English version was not available.
iii TN: My translation. English version was not available.
${ }^{\text {iv }} \mathrm{TN}$ : My translation. English version was not available.
${ }^{\mathrm{v}}$ TN: My translation. English version was not available.
${ }^{\text {vi }}$ TN: My translation. English version was not available.


[^0]:    ${ }^{1}$ Perhaps it was David Ricardo (1817, pp. 6-7) who promoted this belief by commenting on Say's work as follows: "[Say] not only was the first, or among the first, of continental writers, who justly appreciated and applied the principles of Smith, and who has done more than all other continental writers taken together, to recommend the principles of that enlightened and beneficial system to the nations of Europe; but who has succeeded in placing the science in a more logical, and more instructive order; and has enriched it."
    ${ }^{2}$ Napoleon banned the dissemination of Say's book.

[^1]:    ${ }^{4}$ Say calls the three factors of production labor, capital, and land.

[^2]:    ${ }^{5}$ Recherelies sur le principes mathématiques de la théorie des richesses, publised in 1838 . There is a Spanish translation with the title Investigación acera de los principios matemáticos de la teoría de las riquezas por Alianza Editorial (1969).
    ${ }^{6}$ De utilité et de samesure its a collection of articles published since 1844.
    ${ }^{7}$ Entwickehing der Gesetze des niensclilichen Verkelirs nnd der daraus jiiessenden Regeln für menschliches Handeln, published in 1854. A detailed exposition of Gossen's contributions can be found in Kauder (IV, V 1965).

[^3]:    ${ }^{8}$ For more details, see Part I of this essay.

[^4]:    ${ }^{9}$ In a footnote, Menger adds: "Aristotle (De Anima iii.10. 433a 25-38) already distinguished between true and imaginary goods according to whether the needs arise from rational deliberation or are irrational."

[^5]:    ${ }^{10}$ Remember some paragraphs from the classics. Adam Smith (1978, p.358) says: " 1 st, a thing of no use, as (a) lump of clay, brought into the market will give no price, as no one demands it. If it be useful the price will be regulated according to demand, as this use is general or not, and the plenty is to supply it."

[^6]:    David Ricardo (1951, pp. 247-248): "A commodity must be useful to have value but the difficulty of its production is the true measure of its value. For this reason Iron though more useful is of less value than gold."
    John Stuart Mill (1848, p. 442): "That a thing may have any value in exchange, two conditions are necessary. It must be of some use; that is (as already explained), it must conduce to some purpose, satisfy some desire. No one will pay a price, or part with anything which serve some of his purposes, to obtain a thing which serves none of them. But, secondly, the thing must not only have some utility, there must also be some difficulty in its attainment."
    Really, I do not see any difference between Burlamaqui's words and these words by Smith, Ricardo, and Mill. Once again, it seems that Walras did not do a careful reading of the classics.

[^7]:    ${ }^{11}$ As cited by Jaffé (1935, p. 187): "Je ne suis un économiste. Je suis un architecte. Mais je sais mieux l'économie politique que les économistes."

[^8]:    ${ }^{12}$ Published first in The Annals of the American Academy of Political and Social Sciences, Philadelphia, Pennsylvania, in January 1891.
    ${ }^{13}$ With original German title Zum Abschluss des Marxschen Systems, originally in StaatswissenschaftlicheArbeiien - Feslgabenfür Karl Knies zur Fünfuncl.siebzigsien Wieclerkehr, Hearing, Berlin, in 1896.
    ${ }^{14}$ With original German title Per lelzte Masstab der Güterwertes, originally published in Zeilschriftfür Volkswirlschafl und Venvaltung (Vol. III), Vienna, Austria, in 1894.

[^9]:    ${ }^{15}$ For a more complete explanation see Robinson (1934)
    ${ }^{16}$ In particular, see Friedman (1953, Chapter 1).

[^10]:    ${ }^{17}$ For instance, see Nikaido (1978, p. 170 italics added): "The (competitive) firm cannot exert appreciable influence on the market through artificial procedures such as price manipulation since its size is minuscule compared to the economy as a whole. Therefore, this company considers prices as data that it cannot alter by its own means, that is, as restrictions given when establishing its production plan." [TN: My translation. English version was not available.]

